Simple Description of Doublet Bands with Mass of Approximately 100

Koji HIGASHIYAMA$^{1,*}$ and Naotaka YOSHINAGA$^{2,**}$

$^{1}$Department of Physics, Chiba Institute of Technology, Narashino 275-0023, Japan
$^{2}$Department of Physics, Saitama University, Saitama 338-8570, Japan

(Received December 17, 2007)

A simple collective model is applied to understand the structure of the $\Delta I = 1$ doublet bands with the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration in the doubly-odd nuclei, $^{98-104}$Tc and $^{100-106}$Rh. In the model, a doubly-odd nucleus is realized when there is one neutron in the $0h_{11/2}$ orbital and one proton in the $0g_{9/2}$ orbital, and the collective core represents the even-even part of the nucleus. The calculation reproduces the energy spectra and the ratios $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ for the doublet bands quite well. The analysis of the wave functions reveals new band structure, showing various angular momentum configurations of the neutron and the proton, weakly coupled with the quadrupole collective excitations of the core.

§1. Introduction

Medium and heavy doubly-odd nuclei are of special interest owing to the appearance of $\Delta I = 1$ doublet bands built on the high-$j$ orbitals of a neutron and a proton. In these doublet bands, two states with the same spin and parity are observed to be almost degenerate in energy. Such pairs of bands built on the $\nu h_{11/2} \otimes \pi h_{11/2}$ configuration have been experimentally found in many doubly-odd nuclei in the region of mass $A \sim 130$.\cite{1}–\cite{3} Previously these bands were interpreted to be a manifestation of the chiral doublet bands.\cite{4} However, the results of many recent experiments and analyses do not support this interpretation.\cite{5}–\cite{9}

The doublet bands with the $\nu h_{11/2} \otimes \pi h_{11/2}$ configuration in the region of mass $A \sim 130$ were investigated theoretically in the framework of the pair-truncated shell model (PTSM).\cite{10}–\cite{12} In the model, the collective nucleon pairs with angular momenta zero ($S$) and two ($D$) are assumed to be the building blocks for even-even nuclei. Additional unpaired neutron and proton are added to the even-even nuclear states in the description of doubly-odd nuclei. The PTSM calculations were performed for the even-even nuclei in the region of mass $A \sim 130$, and excellent agreements with experimental data were achieved for both energy levels and electromagnetic properties.\cite{13} In its application to doubly-odd nuclei, the same set of interactions as used for the even-even nuclei was simply applied. The theoretical results agree well with the experimental energy spectra and the ratios $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ for the doublet bands. Through the analysis of the PTSM wave functions, the doublet bands are found to be constructed of weak

$^{*}$ E-mail: koji.higashiyama@it-chiba.ac.jp
$^{**}$ E-mail: yoshinaga@phy.saitama-u.ac.jp
couplings of various angular momentum configurations of an unpaired neutron and an unpaired proton, i.e., chopsticks configurations, with the quadrupole collective excitations of the even-even part of the nucleus.

Recently, a much simpler model, the quadrupole coupling model (QCM), was proposed to describe doubly-odd nuclei.\(^{14,15}\) In the model, the core representing the even-even part of the nucleus couples with a neutron and a proton in the high-\(j\) intruder orbitals through a quadrupole-quadrupole interaction, which is assumed to be the most important interaction for understanding the band structure of doublet bands. In some sense, the QCM can be regarded as a simplified version of the interacting boson fermion-fermion model (IBFFM).\(^ {16-18}\) In the IBFFM, nuclear collective excitations of the even-even core are described by bosons, and no assumption is made about the core deformation. The IBFFM provides a powerful tool for the study of doubly-odd nuclei, but it has practical difficulties when determining numerous parameters of the effective interactions. Such difficulties are absent in the QCM, which has only five adjustable parameters for the energy and three (five) parameters for the electric (magnetic) transitions. Theoretical investigations of the doublet bands in the region of mass \(A \approx 130\) were carried out in terms of the QCM.\(^ {14,15}\) The model reproduced experimental energy spectra and features of electromagnetic transitions with good accuracy. By analyzing the QCM wave functions, the excitation mechanism predicted on the basis of the previous PTSM calculations\(^ {10-12}\) has been reconfirmed.

The doublet bands in many doubly-odd nuclei have also been observed for the region of mass \(A \approx 100\).\(^ {19-21}\) In this region the \(\Delta I = 1\) doublet bands are built on the \(\nu h_{11/2} \otimes \pi g_{9/2}\) configuration, and observed as the yrast and yrare bands with negative parity. These nuclei also show even-odd staggering of the ratios \(B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)\). In this paper, we apply the QCM to doubly-odd nuclei, \(^{98-104}\)Tc and \(^{100-106}\)Rh. The calculated energy levels and electromagnetic transitions are compared with the corresponding experimental data. In order to understand the underlying physics, we analyze the QCM wave functions.

The paper is organized as follows. In §2, we present the general framework of the QCM. In §3, numerical results of the energy levels and electromagnetic transitions are given for the doubly-odd nuclei, \(^{98-104}\)Tc and \(^{100-106}\)Rh. In §4, we clarify the structure of the doublet bands through their wave functions. In §5, we give a summary and discussion. In the Appendix, we give explicit expressions for important physical quantities in a two-particle state.

\section{Framework}

The structure of doubly-odd nuclei is described in the framework of the QCM with the coupling of a collective core, representing the even-even part of the nucleus, to one neutron and one proton outside the collective core. The two-particle state with angular momentum \(L\) is expressed as \(|j_\nu j_\pi; L\rangle\), where \(j_\nu\) and \(j_\pi\) represent the single-particle orbitals for the neutron and the proton, respectively. In the present version of the model, the collective core is assumed to be constructed of only the ground-band states of the corresponding even-even nucleus. The core state with
angular momentum $R$ is denoted as $|R\rangle$. Using the core state and the two-particle state, we give the normalized basis wave function of any doubly-odd nucleus,

$$|\Phi(RL; I)\rangle = |R\rangle \otimes |j_{\nu}j_{\pi}; L\rangle^{(I)}$$

(2.1)

where $I$ represents the total spin of the doubly-odd nucleus.

The QCM Hamiltonian is given by

$$H = H_{\text{core}} + H_{\text{cv}} + H_{\text{cp}} + H_{\nu\pi},$$

(2.2)

where $H_{\text{core}}$, $H_{\text{cv}}$, $H_{\text{cp}}$, and $H_{\nu\pi}$ represent the Hamiltonian of the core, the interaction between the core and the neutron, that between the core and the proton, and that between the neutron and the proton, respectively. The Hamiltonian of the collective core, $H_{\text{core}}$, is determined via matrix elements as

$$\langle R' | H_{\text{core}} | R \rangle = E_{R}\delta_{RR'},$$

(2.3)

where $E_{R}$ is the eigenenergy corresponding to the normalized eigenstate $|R\rangle$. We assume, for simplicity, that the other interactions, $H_{\text{cv}}$, $H_{\text{cp}}$, and $H_{\nu\pi}$, are of pure quadrupole-quadrupole types,

$$H_{\text{cv}} = \kappa_{\nu}Q_{\nu} \cdot Q_{\nu},$$

(2.4)

$$H_{\text{cp}} = \kappa_{\pi}Q_{\pi} \cdot Q_{\pi},$$

(2.5)

$$H_{\nu\pi} = \kappa_{\nu\pi}Q_{\nu} \cdot Q_{\pi},$$

(2.6)

where $\kappa$ is the interaction strength, and $Q_{\nu}$, $Q_{\nu}$, and $Q_{\pi}$ indicate the quadrupole operators for the core, the neutron and the proton, respectively.

The Hamiltonian in Eq. (2.2) is diagonalized in terms of the basis functions in Eq. (2.1) as

$$H |\Phi(I\eta)\rangle = E_{I\eta} |\Phi(I\eta)\rangle,$$

(2.7)

where $|\Phi(I\eta)\rangle$ and $E_{I\eta}$ are normalized eigenvectors and eigenenergies, respectively, and $\eta$ is an additional quantum number required to completely specify the state. The normalized eigenvector $|\Phi(I\eta)\rangle$ is linearly expressed in terms of the basis wave functions $|\Phi(RL; I)\rangle$ as

$$|\Phi(I\eta)\rangle = \sum C_{RL}^{I\eta} |\Phi(RL; I)\rangle,$$

(2.8)

where $C_{RL}^{I\eta}$ represents the amplitude of $|\Phi(RL; I)\rangle$ in the state $|\Phi(I\eta)\rangle$. All the details related to the framework of the QCM are given in Refs. 14) and 15).

**§3. Numerical results**

3.1. Interaction strengths for doubly-odd nuclei

In the present study, we apply the QCM to the doubly-odd nuclei, $^{98-104}$Tc and $^{100-106}$Rh, with the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration. From the shell-model point of view, the doubly-odd nuclei in this region have several neutron particles and several
proton holes with respect to the doubly-closed shell of $N = Z = 50$. Thus for the description of the two-particle state, we treat one neutron as a particle and one proton as a hole. Only the $0h_{11/2}$ orbital for the neutron and only the $0g_{9/2}$ orbital for the proton are considered in the present study. The angular momentum and parity of the two-particle state take the values of $L^z = 1^-, 2^-, 3^-, \ldots, 10^-$. In calculating the doubly-odd nuclei with odd $N$ neutrons and odd $Z$ protons, the collective core is considered to have $N-1$ neutrons and $Z+1$ protons. For simplicity, the core is assumed to be made of only the ground-band states. The core angular momentum is restricted to be $R = 0, 2, 4, 6, 8, 10, \text{ and } 12$, since we focus our attention on the low-lying states for the doubly-odd nuclei.

The main difference between the present calculation for the region of mass $A \sim 100$ and the previous calculations for the region of mass $A \sim 130^{14),15}$ should be mentioned. In the previous QCM calculations, the two-particle state is constructed by one neutron hole and one proton particle both in the the same $0h_{11/2}$ orbital. The angular momentum and parity of the two-particle state take values of $0^+, 1^+, 2^+, \ldots, 11^+$. Then, the total basis function of the doubly-odd nuclei has positive parity. On the contrary, in the present calculation, the neutron is treated as a particle, and the proton, as a hole. Since the proton orbital, $0g_{9/2}$, differs from that in the region of mass $A \sim 130$, the total basis function has negative parity. Furthermore, the total spins for the parallel configuration of the neutron and proton angular momenta are different by one unit in spin in these two regions. Namely, the total spin is eleven in the region of mass $A \sim 130$, while it is ten in the region of mass $A \sim 100$.

Table I. Adopted structure coefficients $a$ and $b$, and experimental energy ratios $R_{4/2}$.

<table>
<thead>
<tr>
<th>Nuclear</th>
<th>$a$</th>
<th>$b$</th>
<th>$R_{4/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{100}$Pd</td>
<td>0.31</td>
<td>0.009</td>
<td>2.13</td>
</tr>
<tr>
<td>$^{102}$Pd</td>
<td>0.22</td>
<td>0.020</td>
<td>2.29</td>
</tr>
<tr>
<td>$^{104}$Pd</td>
<td>0.22</td>
<td>0.023</td>
<td>2.38</td>
</tr>
<tr>
<td>$^{106}$Pd</td>
<td>0.20</td>
<td>0.022</td>
<td>2.40</td>
</tr>
<tr>
<td>$^{98}$Ru</td>
<td>0.29</td>
<td>0.013</td>
<td>2.17</td>
</tr>
<tr>
<td>$^{100}$Ru</td>
<td>0.22</td>
<td>0.019</td>
<td>2.27</td>
</tr>
<tr>
<td>$^{102}$Ru</td>
<td>0.18</td>
<td>0.020</td>
<td>2.33</td>
</tr>
<tr>
<td>$^{104}$Ru</td>
<td>0.12</td>
<td>0.021</td>
<td>2.48</td>
</tr>
</tbody>
</table>

In the first step of the QCM calculation, the Hamiltonian of the collective core $H_{\text{core}}$ is fixed to reproduce the energy spectra in the corresponding even-even nucleus. The excitation energies of the core, $E_R$, are given as

$$E_R = aR + bR(R + 1),$$

(3.1)

where $a$ and $b$ are the structure coefficients to be determined experimentally. The structure coefficients $a$ and $b$ are determined so as to reproduce the experimental excitation energies of the $2^+_1, 4^+_1$ and $6^+_1$ states in the ground-state band for each even-even nucleus with $N-1$ neutrons and $Z+1$ protons. The adopted coefficients are listed in Table I. The experimental energy ratios $R_{4/2} (= E_4/E_2)$ are also given in Table I. In Fig. 1, the excitation energies of the cores are compared with experimental data for even-even $^{44}$Ru, and $^{46}$Pd isotopes up to spin 10. It is seen that the calculated levels nicely fit the experimental data for the $2^+_1, 4^+_1, 6^+_1$, and $8^+_1$ states.

In the second step of our calculation, the quadrupole-quadrupole interactions are determined for doubly-odd nuclei. In calculating total energies, the reduced matrix element of the quadrupole operator for the neutron and the proton in a $j$ orbital,
Fig. 1. Calculated energy levels (filled squares), and the experimental data (open circles) as a function of neutron number $N$ for the yrast states ($2^+_1$, $4^+_1$, $6^+_1$, $8^+_1$, and $10^+_1$) for Ru and Pd isotopes. Experimental data were taken from Refs. 22–29).

$Q_{\tau}$ ($\tau = \nu$ or $\pi$), is taken to be

$$\langle j | Q_{\tau} | j \rangle = 1.$$  \hspace{1cm} (3.2)

Since we deal with only the single-$j$ orbitals for both the neutron and the proton, all the dependences on quantum numbers $n$, $\ell$, and $j$ can be absorbed in the interaction strengths, $\kappa_{\nu\nu}$, $\kappa_{\nu\pi}$, and $\kappa_{\pi\pi}$. The quadrupole operator for the core, $Q_c$, is defined through the reduced matrix element as

$$\langle R' | Q_c | R \rangle = \sqrt{(2R' + 1)(2R + 1)} \langle R0R'0 | 20 \rangle ,$$  \hspace{1cm} (3.3)

where $\langle R0R'0 | 20 \rangle$ stands for a Clebsch-Gordan coefficient.

In the previous QCM calculations for the mass $A \sim 130$ nuclei,\textsuperscript{14,15} a linear dependence of the interaction strengths on the valence neutron and proton numbers was introduced. Their strengths were determined from the energy spectra and the ratios $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ for a wide range of doubly-odd nuclei. Unfortunately, the $B(M1)/B(E2)$ ratios have been measured only for a few nuclei in the region of mass $A \sim 100$. The interaction strengths are assumed to be the same for all nuclei in this study. The determined values of parameters (in MeV) are $\kappa_{\nu\nu} = -1.10$, $\kappa_{\nu\pi} = -0.10$, and $\kappa_{\pi\pi} = +6.00$.

To confirm the validity of the present approach, we compare these interaction strengths with those for the nuclei in the region of mass $A \sim 130$. The interaction strength between the proton particle and the core, $\kappa_{cp}$, that between the neutron hole and the core, $\kappa_{ch}$, and that between the neutron hole and the proton particle, $\kappa_{hp}$ correspond to $\kappa_{\nu\nu}$, $\kappa_{\nu\pi}$, and $\kappa_{\pi\pi}$, respectively. The mean values of the interaction strengths were $\kappa_{cp} = -1.55$, $\kappa_{ch} = -0.25$, and $\kappa_{hp} = +6.00$, which are close to the above values in the present calculation.
3.2. Spectra

Using the interaction strengths determined above, the energy spectra are obtained for the doubly-odd nuclei, $^{98-104}$Tc and $^{100-106}$Rh. In $^{100}$Rh, one $\Delta I = 1$ band with negative parity is experimentally found, but its configuration is not yet identified. In Fig. 2, the experimental energy levels are compared with the theoretical levels of the QCM with the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration. Our calculation reproduces the energy levels of the band quite well.

In Fig. 3, the experimental spectra based on the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration are compared with the QCM calculations for $^{106}$Rh and $^{100}$Tc. In the following, the subscripts 1 and 2 denote the yrast states and the yrare states, respectively. In the experiment, the $6^{-}_1$ states appear lower in energy than the $8^{-}_1$ states for both nuclei, but in theory, the $6^{-}_1$ states are predicted to be higher in energy than the $8^{-}_1$ states. For the yrast states with spins greater than 7, calculated energy levels are in excellent agreement with experimental data. Concerning the yrare states, the QCM calculation reproduces the observed levels at the correct positions, except that some states are not observed in the experiment.

In Figs. 4 and 5, the theoretical and experimental energy spectra for $^{102}$Rh, $^{104}$Rh and $^{98}$Tc are given. Concerning $^{104}$Rh, the yrast and yrare bands are experimentally assigned to be built on the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration. The model reproduces the experimental energy levels for both bands quite well. For $^{102}$Rh and $^{98}$Tc, the QCM reproduces the energy levels of the yrast states well. There is no experimental
evidence for the yrare states, but the theoretical yrare states appear almost at the same positions as in the cases of $^{104}$Rh, $^{106}$Rh, and $^{100}$Tc. In Fig. 5, we show the theoretical energy levels of the yrast and yrare states for $^{102}$Tc, and $^{104}$Tc, which are not observed in the experiment.

Fig. 3. Experimental energy levels (expt.) and the QCM results (QCM) for $^{106}$Rh and $^{100}$Tc. Two level sequences of $\Delta I = 1$ on the right represent the yrast band, and two level sequences of $\Delta I = 1$ on the left, the yrare band. Experimental data were taken from Refs. 20) and 21).

Fig. 4. The same as Fig. 3 for $^{102}$Rh and $^{104}$Rh. Experimental data were taken from Refs. 19) and 31).
3.3. Electromagnetic transitions

The $E2$ transition operator is defined as

$$T(E2) = e_c Q_c + e_{\nu} Q_{\nu} + e_{\pi} Q_{\pi},$$

(3.4)

where $e_c$, $e_{\nu}$, and $e_{\pi}$ represent the effective charges of the core, the neutron, and the proton, respectively. The effective charge of the core, $e_c$, is determined by the $B(E2; 0^+_1 \rightarrow 2^+_1)$ value of the even-even nucleus as

$$|e_c| = \sqrt{B(E2; 0^+_1 \rightarrow 2^+_1)/5}.$$ (3.5)

The $B(E2; 0^+_1 \rightarrow 2^+_1)$ values of the cores are determined to reproduce the experimental ones for even-even nuclei, $^{98-104}$Ru and $^{100-106}$Pd. The adopted values are assumed to be smoothly changed as functions of the neutron number $N$ and the proton number $Z$ of the even-even nucleus. The determined functional dependence is given as follows (in $e^2b^2$):

$$B(E2; 0^+_1 \rightarrow 2^+_1) = 0.06(N - 50) + 0.04(50 - Z) - 0.07.$$ (3.6)

Note that in calculating the $E2$ transitions of the doubly-odd nucleus $(N, Z)$, we use the $B(E2; 0^+_1 \rightarrow 2^+_1)$ values of the even-even nucleus $(N - 1, Z + 1)$. In Fig. 6, the calculated $B(E2; 0^+_1 \rightarrow 2^+_1)$ values of the cores are compared with the experimental values for even-even Ru and Pd isotopes. The sign of $e_c$ is determined to reproduce the $B(M1)/B(E2)$ ratios, and taken to be positive for all the doubly-odd nuclei.

The quadrupole operator used for the calculation of the $E2$ transitions is different from that of the energy spectra. The reduced matrix element of the quadrupole operator for a neutron single particle or proton single hole is given by

$$\langle j \parallel Q \parallel j \rangle = \langle n\ell \parallel r^2 \parallel n\ell \rangle \langle j \parallel Y^{(2)} \parallel j \rangle,$$ (3.7)
Fig. 6. Calculated $B(E2; 0^+_1 \rightarrow 2^+_1)$ values (filled squares), and the experimental data (open circles) as a function of neutron number $N$ for even-even Ru and Pd isotopes. Experimental data were taken from Ref. 33).

where

$$\langle n\ell | r^2 | n\ell \rangle = \frac{1}{b^2} \left( 2n + \ell + \frac{3}{2} \right), \quad (3.8)$$

$$\langle j || Y^{(2)} || j \rangle = \sqrt{\frac{5(2j + 1)}{4\pi}} \left( j_{1/2} \right)_{20} \left( j_{1/2} \right)_{20}. \quad (3.9)$$

Here, the harmonic oscillator basis states with the oscillator parameter $b = 1.005A^{1/6}$ fm are used. The effective charges for the neutron particle and the proton hole are fixed to be $e_\nu = +1.0 \; e$ and $e_\pi = -2.0 \; e$. Note that the proton effective charge is chosen to be negative, since the proton is treated as a hole.

In order to check the dependence on the effective charges for the neutron and the proton, we intentionally carry out another calculation with the charges $e_\nu = 0.0 \; e$ and $e_\pi = -1.0 \; e$ for $^{106}$Rh. Figure 7 shows the comparison of the theoretical $B(E2; I \rightarrow I - 2)$ values using two different sets of effective charges: (1) $e_\nu = +1.0 \; e$ and $e_\pi = -2.0 \; e$, and (2) $e_\nu = 0.0 \; e$ and $e_\pi = -1.0 \; e$. It is noted in the figure that these two results agree very well with each other. The $E2$ transitions are not greatly affected by the effective charges of the neutron and the proton.

The $M1$ transition operator is defined in units of $\mu_N$ as

$$T(M1) = \sqrt{\frac{3}{4\pi}} \left( g_c \hat{\ell} + g_{\ell\nu} \ell + g_{s\nu} s_{\nu} + g_{\ell\pi} \ell + g_{s\pi} s_{\pi} \right), \quad (3.10)$$

where $g_c$ stands for the $g$ factor of the core, and $g_{\ell\nu}$ ($g_{s\nu}$) and $g_{\ell\pi}$ ($g_{s\pi}$) represent the $g$ factors for the orbital angular momentum (spin) for the neutron and the proton,
Fig. 7. Theoretical $B(E2; I \rightarrow I - 2)$ values between yrast states (left panel) and between yrare states (right panel) for $^{106}$Rh. Filled circles are for (1) $e_\nu = +1.0$ e and $e_\pi = -2.0$ e, and open squares are for (2) $e_\nu = 0.0$ e and $e_\pi = -1.0$ e. Solid lines indicate the transitions between odd-spin states, and dotted lines, between even-spin states.

respectively. The operators $\hat{R}$, $\ell$, and $s$ represent core angular momentum, orbital angular momentum of the neutron and proton, and spin of the neutron and proton, respectively.

The $g$ factor of the core is determined using the magnetic dipole moment $[\mu(2^+_1)]$ of the $2^+_1$ state in the neighboring even-even nuclei with $N-1$ neutrons and $Z+1$ protons as

$$g_c = \frac{\mu(2^+_1)}{2},$$

where the adopted dipole moment is $+0.85 \ \mu_N$ for all the nuclei; namely, it is the mean value of the experimental data of the six even-even nuclei $^{98-104}$Ru and $^{102,104}$Pd. The $g$ factors for the neutron and proton are adopted to be $g_{\ell\nu} = 0.00$, $g_{\ell\pi} = 1.00$, $g_{s\nu} = -2.68$, and $g_{s\pi} = 3.91$, which are the same values as used in the QCM calculations for the region of mass $A \sim 130$. All the details of the calculations of $E2$ and $M1$ transitions are given in the appendixes of Ref. 15).

In Fig. 8, the theoretical ratios $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ for the yrast states of doubly-odd nuclei, $^{98-102}$Tc and $^{100-104}$Rh, are compared with experimental data. For $^{106}$Rh, our result gives a successful description of the large-amplitude staggering of the $B(M1)/B(E2)$ ratios. In $^{100}$Tc, the theoretical ratios are slightly larger compared with experimental data, but the staggering pattern is in phase. For $^{102}$Rh and $^{104}$Rh, the even-odd staggering of the $B(M1)/B(E2)$ ratios (not shown in the figure) has been experimentally found. The present calculation supports this experimentally observed behavior. For $^{100}$Rh, $^{98}$Tc, and $^{102}$Tc, the staggering behavior of the $B(M1)/B(E2)$ ratios is similar to that in the cases of $^{102}$Rh, $^{104}$Rh, $^{106}$Rh, and $^{100}$Tc.

The staggering feature of the $B(M1)/B(E2)$ ratios differs from that given by the previous QCM calculations. In the present study, the $B(M1)/B(E2)$ ratios between the yrast states ($I \geq 10$) are large for the transitions from the even-spin states, and small for those from the odd-spin states. In contrast, in the region of mass $A \sim 130$, the $B(M1)/B(E2)$ ratios from the odd-spin states are larger than those from the even-spin states. This is due to the difference in angular momenta between
the proton hole orbital \(0g_{9/2}\) for the present calculation and the proton particle orbital \(0h_{11/2}\) for the previous calculations. As discussed in §4.1, the staggering of the \(B(M1)/B(E2)\) ratios is caused by two-particle motion of the neutron and the proton along the yrast line. Since the proton orbitals are different between for the region of mass \(A \sim 100\) and for the region of mass \(A \sim 130\) by one unit in spin, the behavior of the staggering patterns is opposite to one another.

In the case of the region of mass \(A \sim 130\), the appearance of the staggering feature of the \(B(M1)/B(E2)\) ratios is in a good correlation with the strengths of the quadrupole-quadrupole interaction between the core and the neutron hole. The staggering feature disappears when the absolute strength exceeds 1.5. It is also found that there is an evident correlation between the staggering and the energy ratio \(R_{4/2}\).
If the ratio $R_{4/2}$ is larger than 2.5, there is less staggering. In the case of the region of mass $A \sim 100$, the ratio $R_{4/2}$ is smaller than 2.5 for all nuclei (as shown in Table I). The theoretical result shows the even-odd staggering of the $B(M1)/B(E2)$ ratios.

In Fig. 9, the calculated $B(E2; I \rightarrow I - 2)$ values for the yrast and the yrare states of $^{106}$Rh and $^{100}$Tc are shown as functions of spin $I$. The strong $E2$ transitions connect the yrast states with spin $I$ ($I \geq 11$) to the yrast states with spin $I - 2$. In contrast, the $B(E2)$ values are found to be small from the yrast states to the yrare states ($I \geq 11$). Thus, the strong $E2$ transitions for the yrast states indicate that the odd-spin and the even-spin states respectively form two $\Delta I = 2$ $E2$ bands starting from the bandhead states of $9^+_1$ and $10^-_1$.

For the yrare states, the odd-spin states ($I \geq 11$) and the even-spin states ($I \geq 10$) are respectively linked by the strong $E2$ transitions. Concerning the transitions between the yrast and yrare states, the $B(E2; 10^-_1 \rightarrow 8^+_1)$ and $B(E2; 9^-_2 \rightarrow 7^-_1)$ values are the largest compared with those of other transitions for both $^{106}$Rh and $^{100}$Tc. For the other transitions (not shown in the figure), large $B(E2)$ values are predicted for $15^-_3 \rightarrow 13^-_3 \rightarrow 11^-_3 \rightarrow 9^-_2$. On the basis of the theoretical $B(E2)$ values, the even-spin states ($8^-_1$, $10^-_2$, $12^-_2$, $14^-_2$, $16^-_2$), the odd-spin states ($7^-_1$, $9^-_2$, $11^-_3$, $13^-_2$, $15^-_3$), and the odd-spin yrare states ($I \geq 11$) respectively form three $\Delta I = 2$ $E2$ bands with their bandhead states of $8^-_1$, $7^-_1$ and $11^-_3$. We conclude that the following members form five $\Delta I = 2$ $E2$ bands, each starting from the first member as the bandhead state: (1) $10^-_1$, $12^-_1$, $14^-_1$, $16^-_1$, $18^-_1$, (2) $9^-_1$, $11^-_1$, $13^-_1$, $15^-_1$, $17^-_1$, (3) $8^-_1$, $10^-_2$, $12^-_2$, $14^-_2$, $16^-_2$, (4) $7^-_1$, $9^-_2$, $11^-_3$, $13^-_2$, $15^-_3$, and (5) $11^-_2$, $13^-_2$, $15^-_2$, $17^-_2$. 
In Fig. 10, the calculated $B(M1; I \rightarrow I - 1)$ values for the yrast and the yrare states of $^{106}$Rh and $^{100}$Tc are shown as functions of spin $I$. For both nuclei, the $B(M1)$ values between the yrast states ($I \geq 10$) are large for the transitions from the even-spin states to the even-spin states, and small for those from the odd-spin states to the even-spin states. On the contrary, both $B(M1)$ values are found to be very small for the yrare states ($I \geq 11$). This means that the yrast and yrare states have very different structures, and there is less of a relationship between the $\Delta I = 1$ yrast and yrare bands. With respect to other $M1$ transitions, large $B(M1)$ values are predicted for the $9^-_1 \rightarrow 8^-_1 \rightarrow 7^-_1$, $11^-_1 \rightarrow 10^-_2 \rightarrow 9^-_2$, $13^-_1 \rightarrow 12^-_2 \rightarrow 11^-_3$, $15^-_1 \rightarrow 14^-_2 \rightarrow 13^-_3$, and $17^-_1 \rightarrow 16^-_2 \rightarrow 15^-_3$ transitions (the $12^-_2 \rightarrow 11^-_3$, $14^-_2 \rightarrow 13^-_3$, and $16^-_2 \rightarrow 15^-_3$ transitions are not shown in the figure). The strong $M1$ transitions indicate that the $\Delta I = 1$ $M1$ bands are composed of the following five level sequences: (a) $7^-_1$, $8^-_1$, $9^-_1$, $10^-_1$, (b) $9^-_2$, $10^-_2$, $11^-_2$, $12^-_2$, (c) $11^-_3$, $12^-_2$, $13^-_2$, $14^-_2$, (d) $13^-_3$, $14^-_2$, $15^-_1$, $16^-_1$, and (e) $15^-_3$, $16^-_2$, $17^-_1$, $18^-_1$.

3.4. $E2$ and $M1$ band structure

The partial level scheme of $^{106}$Rh is shown in Fig. 11. We do not display figures for other nuclei, but similar level schemes are deduced also for the doubly-odd nuclei, $^{98-102}$Tc and $^{100-104}$Rh. The QCM gives five $\Delta I = 2$ $E2$ bands starting from the bandhead states of $7^+_1$, $8^+_1$, $9^+_1$, $10^+_1$, and $11^-_2$. The states within bands are connected by the strong $E2$ transitions to the same members of the $\Delta I = 2$ $E2$ bands, and by the strong $M1$ transitions to the states in the neighboring $\Delta I = 2$ $E2$ bands. These
Fig. 11. Partial level scheme of $^{106}$Rh suggested by the QCM calculation. Green arrows indicate $E2$ transitions ($B(E2) \geq 0.02 e^2 b^2$), and orange arrows denote $M1$ transitions ($B(M1) \geq 0.40 \mu_N^2$). Numerals on the right side of $E2$ transitions denote $B(E2)$ values (in $10^{-2} e^2 b^2$), and those beneath $M1$ transitions denote $B(M1)$ values (in $\mu_N^2$). Schematic illustrations of chopsticks configurations are presented below each $\Delta I = 2$ $E2$ band. Five $\Delta I = 2$ $E2$ bands have the configurations shown by the schematic illustrations at the bottom. Red and blue arrows indicate the angular momenta of the neutron and the proton for the bandhead state, respectively. Their angles are given in the geometrical (classical) picture. They are different from the quantum-mechanical angles, i.e., the effective angles, given in Table IV in the Appendix. The vector sum of two angular momenta is represented by black arrows.

strong $M1$ transitions give five $\Delta I = 1$ $M1$ bands. In this figure, the $\Delta I = 2$ $E2$ bands are vertically formed, whereas the $\Delta I = 1$ $M1$ bands are horizontally formed. It should be noted that the structure of the odd-spin yrare states ($11^{-}_{2}$, $13^{-}_{2}$, $15^{-}_{2}$, and $17^{-}_{2}$) are quite different from those of the other $\Delta I = 2$ $E2$ bands, since these states are not connected by the strong $M1$ transitions to any member of other $\Delta I = 2$ $E2$ bands.

§4. Analysis of QCM wave functions

4.1. Effective angles between proton and neutron angular momenta and squares of core angular momentum

To deepen our understanding of the excitation mechanism of the yrast and yrare states with the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration, we calculate effective angles between two angular momenta of the neutron and the proton, and the squares of the core angular momentum. The effective angle between two angular momenta of the neutron and
the proton, $\theta$, is defined as

$$\cos \theta = \frac{\langle \Phi(I\eta) | j_\nu \cdot j_\pi | \Phi(I\eta) \rangle}{\sqrt{\langle \Phi(I\eta) | j_\nu^2 \Phi(I\eta) \rangle \langle \Phi(I\eta) | j_\pi^2 \Phi(I\eta) \rangle}}$$

(4.1)

where $j_\nu$ and $j_\pi$ represent the angular momentum operators for the neutron and the proton, respectively. The effective angles never become zero because of quantum fluctuations, even though two angular momenta point toward the same direction. Here, we consider the two-particle state which consists of a neutron in the $0h_{11/2}$ orbital and a proton in the $0g_{9/2}$ orbital. The effective angle turns out to be $34^\circ$, $60^\circ$, $79^\circ$, and $94^\circ$ for the two-particle states $|j_\nu j_\pi; L\rangle$ with angular momentum $L = 10, 9, 8, \text{ and } 7$, respectively, as given in Table IV in the Appendix. Since a realistic nuclear state is expressed as a superposition of several configurations, the effective angle becomes even larger than $34^\circ$.

The square of the core angular momentum $\langle \hat{R}^2 \rangle$ is defined as

$$\langle \hat{R}^2 \rangle = \langle \Phi(I\eta) | \hat{R}^2 | \Phi(I\eta) \rangle,$$

(4.2)

where $\hat{R}$ is the core angular momentum operator. For the pure core state $|R\rangle$ with angular momentum $R$, this value is given as $\langle \hat{R}^2 \rangle = R(R+1)$. In the present study, the values (in units of $\hbar^2$) become $\langle \hat{R}^2 \rangle = 0, 6, 20, 42, 72, 110, \text{ and } 156$ for the core angular momenta $R = 0, 2, 4, 6, 8, 10, \text{ and } 12$, respectively.

In Fig. 12, the effective angles $\theta$ and the values of $\langle \hat{R}^2 \rangle$ for the yrast and yrare states of $^{106}$Rh and $^{100}$Tc are shown as functions of spin $I$. Concerning the states with spins less than or equal to 10, the effective angles $\theta$ decrease monotonically as spin $I$ increases. The values of $\langle \hat{R}^2 \rangle$ for these states are nearly zero. In contrast, the yrast and yrare states with spins greater than 10 exhibit the even-odd staggering of the effective angles $\theta$. Furthermore, the staggering is out of phase for the yrast and yrare states. The values of $\langle \hat{R}^2 \rangle$ show staircase-like behavior for both the yrast and yrare lines.

From the behavior of the effective angles $\theta$, the yrast and yrare states can be divided into two spin regions: the states with spins less than or equal to 10, and the states with spins greater than 10. In the following, we discuss the internal structure of the yrast and yrare states.

4.1.1. Yrast states with spins less than or equal to 10

For the $7^-_1$ state two angular momenta of the neutron and the proton are almost perpendicular to one another ($\theta \sim 90^\circ$), as shown in the upper panel of Fig. 12. The effective angles $\theta$ along the yrast line decrease monotonically as spin $I$ goes up to 10. This means that two angular momenta of the neutron and the proton close like a pair of chopsticks as spin increases from 7 to 10.

The values of $\langle \hat{R}^2 \rangle$ for the yrast states with spins between 7 and 10 are almost zero, as seen in the middle panel of Fig. 12, although their values are not exactly zero owing to a small admixture of other cores ($R = 2, 4, \ldots$). It is inferred that the cores in these states have mainly the angular momentum $R = 0$.

Thus, it is summarized, for the yrast states with spins between 7 and 10, that (i) neutron and proton angular momenta close monotonically and (ii) the cores have
Fig. 12. Upper panel: Effective angles $\theta$ of two angular momenta for the yrast and yrare states of $^{106}$Rh and $^{100}$Tc. Solid lines indicate the effective angles for the yrast states, and dotted lines indicate those for the yrare states. Middle panel: Calculated values of $\langle \hat{R}^2 \rangle$ for the yrast and yrare states of $^{106}$Rh and $^{100}$Tc. Solid lines indicate the values of $\langle \hat{R}^2 \rangle$ for the yrast states, and dotted lines indicate those for the yrare states. Lower panel: Effective angles $\varphi_{\tau}$ ($\tau = \nu$ or $\pi$) of two angular momenta for the yrast states of $^{106}$Rh and $^{100}$Tc. Solid and dotted lines indicate the effective angles between the core and the neutron and those between the core and the proton, respectively.

mainly a constant angular momentum $R = 0$. These two findings indicate that the change of the two-particle state gives rise to the large $M1$ transitions between the yrast states ($I \leq 10$) (see Fig. 10).

4.1.2. Yrast states with spins greater than 10

In this spin region, the yrast states exhibit the even-odd staggering of the effective angles $\theta$, whose values are smaller than 60°. Considering the fact that the two-particle state $|j_\nu j_\pi; L\rangle$ with angular momentum $L = 9$ ($\theta = 60^\circ$) has a near parallel configuration of two angular momenta, they point toward almost the same direction at spin $11^-_1$ and subsequently repeat the opening and closing movement with a small difference in their angle, namely, a difference of only one unit in spin. Such two-particle motion is called chopsticks motion.

Concerning the core angular momentum, the values of $\langle \hat{R}^2 \rangle$ for the even-spin yrast states ($I$) ($I = 12, 14, 16, 18$) and the odd-spin yrast states ($I - 1$) are close to
one another, which means that these two states are constructed from the same core state $|\hat{\mathcal{R}}\rangle$. The strong $M1$ transitions from states ($I$) to states ($I-1$) are interpreted to originate from the opening of the two angular momenta of the neutron and the proton by one unit in spin.

On the other hand, the values of $\langle \hat{\mathcal{R}}^2 \rangle$ are different for the even-spin yrast states ($I$) and the odd-spin yrast states ($I+1$). In fact, the values of $\langle \hat{\mathcal{R}}^2 \rangle$ for the $16^+_1$ and $17^+_1$ states in $^{106}$Rh ($^{100}$Tc) are 46 and 74 (47 and 74), respectively, which are close to the values $\langle \hat{\mathcal{R}}^2 \rangle = 42$ and 72 corresponding to the core angular momenta $\mathcal{R} = 6$ and 8. Although the two angular momenta of the neutron and the proton close for the transitions from states ($I+1$) to states ($I$), the $M1$ transitions are basically hindered because the core angular momenta are different by two units in spin. Namely, the $M1$ transition matrix elements are almost zero because the main core angular momenta are different for the initial and final states.

Thus, it is summarized, for the yrast states with spins greater than 10, that (i) neutron and proton angular momenta repeat the opening and closing movement like a pair of chopsticks and (ii) the cores have the same angular momenta for the even-spin yrast states ($I$) and the odd-spin yrast states ($I-1$), while they are different for the even-spin yrast states ($I$) and the odd-spin yrast states ($I+1$). These two findings indicate that strong $M1$ transitions for the yrast states occur from states ($I$) to states ($I-1$), while basically those transitions are hindered from states ($I+1$) to states ($I$).

4.1.3. Even-spin yrare states with spins greater than or equal to 10

As seen from Fig. 10, the large $B(M1)$ values are predicted for the transitions between the odd-spin yrast states ($I-1$) ($I-1 = 11$, 13, 15, and 17) and the even-spin yrare states ($I-2$) ($I-2 = 10$, 12, 14, and 16). These $M1$ transitions are also explained by the effective angles $\theta$ and the values of $\langle \hat{\mathcal{R}}^2 \rangle$. The effective angles $\theta$ for states ($I-2$) are larger than those for states ($I-1$). Since the values of $\langle \hat{\mathcal{R}}^2 \rangle$ are similar for states ($I-2$) and states ($I-1$), the large $M1$ transitions originate only from the opening of the two angular momenta of the neutron and proton.

From the preceding arguments, we conclude that the even-spin yrast state ($I$) ($I = 10$, 12, 14, 16, and 18), the odd-spin yrast state ($I-1$), and the even-spin state ($I-2$) ($8^+_1, 10^+_2, 12^+_2, 14^+_2$, and $16^+_2$), which are members of the $\Delta I = 1 M1$ bands, are basically built on the same core. The $\Delta I = 1 M1$ bands are horizontally formed in Fig. 11. The strong $M1$ transitions along the $\Delta I = 1 M1$ bands are caused by the opening of neutron and proton angular momenta.

4.2. Effective angles between core and nucleon angular momenta

The effective angle between the angular momenta of the core and the neutron or proton, $\varphi_\tau$ ($\tau = \nu$ or $\pi$), is defined as

$$
\cos \varphi_\tau = \frac{\langle \Phi(I\eta) | \hat{\mathcal{R}} \cdot j_\tau | \Phi(I\eta) \rangle}{\sqrt{\langle \Phi(I\eta) | \hat{\mathcal{R}}^2 | \Phi(I\eta) \rangle \langle \Phi(I\eta) | j_\tau^2 | \Phi(I\eta) \rangle}}.
$$

(4.3)

In Fig. 12, the effective angles $\varphi_\tau$ for the yrast states of $^{106}$Rh and $^{100}$Tc are
shown as functions of spin \( I \). Since the values of \( \langle \hat{R}^2 \rangle \) in low-spin states (\( I < 13 \)) are very small compared with those of high-spin yrast states, the denominator of Eq. (4-3) drastically changes for a small admixture of the basis wave functions. Thus, no definitive argument is drawn for the mutual angles of the core and the nucleon angular momenta. With respect to the high-spin states (\( I \geq 13 \)), the effective angles between the two angular momenta of the core and the proton are smaller than \( 38^\circ \) for both nuclei. The effective angles of the core and the neutron show the staggering pattern, whose average angles gradually decrease as spin \( I \) increases. The behavior of the effective angles suggests that the two angular momenta of the core and each nucleon eventually point toward the same direction at high spin. Considering the small effective angles \( \theta \) between the neutron and the proton at high spin, it is concluded that the high-spin states (\( I \geq 13 \)) are built by totally parallel coupling of the three angular momenta of the core, the neutron, and the proton. Similar conclusions are drawn for other doubly-odd nuclei, \(^{98,102}\)Tc and \(^{100-104}\)Rh.

The difference between the chiral scheme and that of the QCM is given as follows. In the chiral picture,\(^4\) the total angular momentum is tilted with respect to the planes defined by the three principal axes, i.e., the short, long, and intermediate axes of the triaxial core. This situation is realized when the angular momenta of the neutron, the proton, and the triaxial core tend to align with the short, long, and intermediate axes, respectively. This indicates that the angular momenta of the core, the neutron, and the proton tend to be perpendicular to each other. However, the physical picture realized in the QCM is far different from the ideal chiral picture for the high-spin states. Firstly, in the QCM calculations, the yrast states with spins greater than 10 have near-parallel configurations of the two angular momenta of the neutron and the proton. Secondly, the high-spin yrast states (\( I \geq 13 \)) are built by totally parallel coupling of the three angular momenta of the core, the neutron, and the proton. Thus the QCM picture for the double bands completely contradicts the chiral picture.

4.3. Major components for QCM wave functions

More detailed information on the structure of the yrast and yrare states can be obtained through an analysis of the wave functions. In the QCM, the eigenstate \( |\Phi(I\eta)\rangle \) is expressed as a linear combination of the basis wave functions \( |\Phi(RL; I)\rangle \) as in Eq. (2-1). Thus the major components of the wave functions provide us with the information of the band structure. We abbreviate the wave function \( |\Phi(RL; I)\rangle \) as \((R, L)\) in the following. In Tables II and III, probabilities \( |C^{I\eta}_{RL}|^2 \) of the components \((R, L)\) for the yrast and yrare states are shown for \(^{106}\)Rh. In Table II it is found that the \( 7^+_1, 8^-_1, 9^+_1, \) and \( 10^-_1 \) states are made mainly of the components \((0,7), (0,8), (0,9), \) and \((0,10), \) respectively. Although the component \((2,9)\) in the \( 10^-_1 \) state is not negligible, we can say that the core with angular momentum \( R = 0 \) plays an important role in describing these yrast states, and that the two-particle states carry a dominant part of the total spin.

Concerning the yrast states with spin greater than 11 (see Table III), the even-spin state \((I)\) \((I = 12, 14, \text{and} 16)\) and the odd-spin state \((I - 1)\) are built on the same angular momentum state of the core \((R = 2, 4, \text{and} 6)\). The even-spin
Table II. Probabilities (in percentage) of the components \((R, L)\) for the \(7_1^-, 8_1^-, 9_1^-, 9_2^-, 10_1^-,\) and \(10_2^-\) states of \(^{106}\text{Rh}\). The main components are indicated in bold font.

<table>
<thead>
<tr>
<th>State</th>
<th>((0,7))</th>
<th>((0,8))</th>
<th>((0,9))</th>
<th>((0,10))</th>
<th>((2,7))</th>
<th>((2,8))</th>
<th>((2,9))</th>
<th>((2,10))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7_1^-)</td>
<td>76</td>
<td>3</td>
<td>9</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8_1^-)</td>
<td>75</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9_1^-)</td>
<td>67</td>
<td>3</td>
<td>14</td>
<td>13</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9_2^-)</td>
<td>5</td>
<td>32</td>
<td>37</td>
<td>17</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10_1^-)</td>
<td></td>
<td>46</td>
<td>9</td>
<td>26</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10_2^-)</td>
<td></td>
<td>19</td>
<td>39</td>
<td>16</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table III. The same as Table II, but for the yrast and yrare states with spins \(I = 11, 12, 13, 14,\) 15, and 16.

<table>
<thead>
<tr>
<th>State</th>
<th>((2,9))</th>
<th>((2,10))</th>
<th>((4,8))</th>
<th>((4,9))</th>
<th>((4,10))</th>
<th>((6,8))</th>
<th>((6,9))</th>
<th>((6,10))</th>
<th>((8,8))</th>
<th>((8,9))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(11_1^+)</td>
<td>57</td>
<td>20</td>
<td>10</td>
<td>9</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(11_2^-)</td>
<td>15</td>
<td>63</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(12_1^+)</td>
<td>45</td>
<td>15</td>
<td>27</td>
<td>9</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(12_2^-)</td>
<td>27</td>
<td>50</td>
<td>6</td>
<td>10</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(13_1^-)</td>
<td>71</td>
<td>14</td>
<td>7</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(13_2^-)</td>
<td>15</td>
<td>72</td>
<td>1</td>
<td>2</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(14_1^+)</td>
<td></td>
<td>70</td>
<td>8</td>
<td>14</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(14_2^-)</td>
<td>19</td>
<td>59</td>
<td></td>
<td>15</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(15_1^-)</td>
<td></td>
<td>80</td>
<td></td>
<td>11</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(15_2^-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(16_1^-)</td>
<td></td>
<td>87</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(16_2^-)</td>
<td></td>
<td>8</td>
<td>65</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

state \((I)\) and the odd-spin state \((I - 1)\) are made of the \(L = 10\) and 9 components, respectively. These results are consistent with the analysis of the effective angles \(\theta\) and the values of \(\langle R^2 \rangle\). Furthermore, it is found that, for the main component of the even-spin yrast state \((I)\), the sum of the three angular momenta of the core, the neutron, and the proton is equal to the total spin. This clearly shows that the three angular momenta point toward the same direction.

It should be reminded that the structure of the odd-spin yrare states \((11_2^-, 13_2^-,\) and \(15_2^-\)) are quite different from those of the other \(\Delta I = 2\) \(E2\) bands. Since the odd-spin yrare states are made mainly of the \(L = 10\) component, they have the fully aligned configuration of two angular momenta of the neutron and the proton. This two-particle configuration is similar to that of the even-spin yrast states \((10_1^-, 12_1^-, 14_1^-,\) and \(16_1^-\)). Comparing the structures of the \(11_2^-), 13_2^-,\) and \(15_2^-\) states with those of the \(12_1^-, 14_1^-,\) and \(16_1^-\) states, it is found that the main component of the even-spin yrast state \((I)\) \((I = 12, 14,\) and \(16)\) and that of the odd-spin yrare state \((I - 1)\) have the same angular momenta of the core \((R)\) and the two-particle state \((L)\). This means that the coupled angular momentum of the core \((R)\) and the two-particle state \((L)\) is different for the even-spin yrast states \((I)\) and the odd-spin yrare states \((I - 1)\) by one unit in spin.

Finally, we focus our attention on the chopsticks configurations made by two angular momenta of the neutron and the proton. Through the above analysis, it
was found that the even-spin yrast states \((I)\) \((I = 10, 12, 14,\) and 16\), the odd-spin yrast states \((I - 1)\), and the even-spin states \((I - 2)\) \((8^-_1, 10^-_2, 12^-_2, 14^-_2,\) and 16^-_2\) are basically built on the two-particle configurations with angular momenta \(L = 10,\) 9, and 8, respectively. Furthermore, the odd-spin yrare states \((11^-_2, 13^-_2,\) and 15^-_2\) have the two-particle configurations with angular momentum \(L = 10\). These single-particle configurations of the two angular momenta of the neutron and the proton are called chopsticks configurations. On the basis of the preceding considerations, the band structure of the \(\Delta I = 1\) \(M1\) and \(\Delta I = 2\) \(E2\) bands is interpreted as arising from a weak coupling of the chopsticks configurations with the quadrupole collective excitations of the even-even part of the nucleus.

In Fig. 11, schematic illustrations of the chopsticks configuration are presented below each \(\Delta I = 2\) \(E2\) band. The members of each \(\Delta I = 2\) \(E2\) band are built mainly from the same chopsticks configuration. This excitation mechanism is essentially the same as that of the previous QCM calculations in the region of mass \(A \sim 130,^{14,15}\) except for the angular momentum and parity of the two-particle state, which arise from the difference in the proton orbital between the \(0g_{9/2}\) orbital for the present calculation and the \(0h_{11/2}\) orbital for the previous calculations.

\section*{§5. Summary and conclusions}

In the present paper, the doublet bands with the \(\nu h_{11/2} \otimes \pi g_{9/2}\) configuration in the doubly-odd nuclei, \(^{98-104}\)Tc and \(^{100-106}\)Rh, were investigated in terms of the quadrupole coupling model (QCM). In the model, the basis wave function is constructed of one neutron in the \(0h_{11/2}\) orbital and one proton in the \(0g_{9/2}\) orbital, and a core representing the collective excitations of the even-even part of the nucleus. The effective Hamiltonian consists of the Hamiltonian of the core, and the quadrupole-quadrupole interaction between the core and the neutron, that between the core and the proton, and that between the neutron and the proton. In a sense, the QCM is very similar to the particle-rotor model (PRM), but has the following different assumptions. (i) In the PRM, the coupling between the particle and the rotor is described by the Coriolis coupling \(I \cdot j_{\tau} (\tau = \nu \text{ or } \pi),\) while it is a quadrupole-coupling \(Q_c \cdot Q_{\tau}\) in the QCM. (ii) In the PRM, a static deformation, either triaxial or not, is assumed for the core, but in the QCM, no assumption is made on the deformation.

We applied the QCM to the doubly-odd nuclei in the region of mass \(A \sim 100.\) For all the nuclei, our calculation reproduced the experimental energy levels of the yrast and the yrare states quite well. Concerning the yrast states of \(^{106}\)Rh and \(^{100}\)Tc, the large-amplitude staggering of the ratios \(B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)\) is experimentally observed. The QCM successfully reproduced the even-odd staggering of the \(B(M1)/B(E2)\) ratios. From the results of the electromagnetic transitions, it was confirmed that the strong \(M1\) transitions connect the even-spin yrast states \((I)\) \((I = 10, 12, 14, 16,\) and 18\) to the odd-spin yrast states \((I - 1)\), the states \((I - 1)\) to the even spin states \((I - 2)\), and the states \((I - 2)\) to the odd spin states \((I - 3)\). They form five \(\Delta I = 1\) \(M1\) bands. The \(E2\) character of the strong transitions indicates that five \(\Delta I = 2\) \(E2\) bands are formed starting from the bandhead states of \(7^-_1, 8^-_1,\)
$9^1, 10^1$, and $11^2$.

In order to clarify the band structure of the doublet bands with the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration, we calculated three kinds of the effective angles and the squares of the core angular momenta. The calculated effective angles showed that the angular momenta of the neutron and the proton, like a pair of chopsticks, close with increasing spin $I$ along the yrast line ($I \leq 10$). For the yrast states with spins greater than 10, the pair of chopsticks opens and closes repetitively as spin $I$ increases (chopsticks motion), causing strong $M1$ transitions between the odd-spin states ($I - 1$) and the even-spin states ($I$). However, there occur very weak transitions between the even-spin states ($I$) and the odd-spin states ($I + 1$) since their core angular momenta are different. For the doubly-odd nuclei, $^{98-102}$Tc and $^{100-106}$Rh, it was concluded that the high-spin states ($I \geq 13$) are built by totally parallel coupling of the three angular momenta of the core, the neutron, and the proton. The behavior of the three angular momenta is further confirmed by examining the probabilities of the components for the QCM wave functions.

The experimentally identified $\Delta I = 1$ doublet bands with the $\nu h_{11/2} \otimes \pi g_{9/2}$ configuration are interpreted as arising from the chopsticks configurations of the two particles, which represent the angular momenta of the neutron and the proton, weakly coupled with the quadrupole collective excitations of the core. The chopsticks configurations and the core produce characteristic $\Delta I = 1$ $M1$ and $\Delta I = 2$ $E2$ bands. The chopsticks motion along the yrast line ($I \geq 9$) is essential for understanding the mechanism of the strong staggering $M1$ transitions.

In summary, the energy levels of the $\Delta I = 1$ doublet bands and the even-odd staggering of the $B(M1)/B(E2)$ ratios in the region of mass $A \sim 100$ are reproduced using the QCM, as in the case of the region of mass $A \sim 130$.\textsuperscript{14,15} For the staggering feature of the $B(M1)/B(E2)$ ratios and angular momenta of the core and the particle, the model predicts that (i) the strong staggering occurs only when the core and the particle are weakly coupled, and (ii) at low spins ($I = 7$ and 8), the two angular momenta of the neutron and the proton are almost perpendicular to one another, while at high spins ($I \geq 13$), the three angular momenta of the core, the neutron, and the proton become totally parallel to each other. We hope future experiments will unveil the secret of the $\Delta I = 1$ doublet bands in more detail, and that our theoretical predictions will be confirmed in the near future.

Acknowledgements

This work was supported by Grants-in-Aid for Scientific Research (19740146) and (20540250) from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

Appendix A

--- Two-Particle State of Neutron and Proton ---

In this appendix, we give the excitation energies, the $M1$ transitions and the effective angles for a two-particle state of one neutron in orbital $j_\nu$ and one proton.
in orbital \( j_\pi \). The wave function of the two-particle state with the total spin \( L \) and its projection \( M \) is written as

\[
|j_\nu j_\pi; LM\rangle = \sum_{m_\nu m_\pi} (j_\nu m_\nu j_\pi m_\pi | LM \rangle | j_\nu m_\nu \rangle | j_\pi m_\pi \rangle
\]

\[
= ([j_\nu] \otimes [j_\pi])^{(L)}_M,
\]

where \( |j_\pi m_\pi \rangle \) (\( \tau = \nu \) or \( \pi \)) denotes a single-particle state and \((j, m)\) represents a set of quantum numbers necessary to specify the state \((n, \ell, j, m)\). Here, we assume \( j_\nu = 11/2, \ell_\nu = j_\nu - 1/2 = 5 \) and \( n_\nu = 0 \) for the neutron intruder orbital \( 0\hbar_{11/2} \), and \( j_\pi = 9/2, \ell_\pi = j_\pi - 1/2 = 4 \) and \( n_\pi = 0 \) for the proton intruder orbital \( 0\hbar_9/2 \).

The matrix element of the quadrupole-quadrupole interaction between the neutron and the proton in Eq. (2-6) is expressed as

\[
\langle j_\nu j_\pi; L' | H_{\nu \pi} | j_\nu j_\pi; L \rangle = \kappa_{\nu \pi} \langle j_\nu j_\pi; L' | Q_{\nu} \cdot Q_{\pi} | j_\nu j_\pi; L \rangle
\]

\[
= \kappa_{\nu \pi} \delta_{L' L} (-1)^{L + j_\nu + j_\pi} \left\{ \begin{array}{ccc} j_\nu & j_\pi & L \\ j_\nu & j_\pi & 2 \end{array} \right\},
\]

where \( \langle j | Q | j \rangle = 1 \) is assumed. The energies are numerically given in Table IV with \( \kappa_{\nu \pi} = 1.0, j_\nu = 11/2, \) and \( j_\pi = 9/2 \). As seen in the table, energies are ordered as \( 7^-, 8^-, 6^-, .. \) from which each band starts as a bandhead state.

The reduced matrix element of the \( M1 \) transition operator in Eq. (3-10) is expressed as

\[
\langle j_\nu j_\pi; L' || T(M1) || j_\nu j_\pi; L \rangle = \sqrt{\frac{3}{4 \pi}} \left[ \langle j_\nu j_\pi; L' || (g_{\ell \nu} \ell_\nu + g_{s \nu} s_\nu) || j_\nu j_\pi; L \rangle + \langle j_\nu j_\pi; L' || (g_{\ell \pi} \ell_\pi + g_{s \pi} s_\pi) || j_\nu j_\pi; L \rangle \right],
\]

where

\[
\langle j_\nu j_\pi; L' || (g_{\ell \nu} \ell_\nu + g_{s \nu} s_\nu) || j_\nu j_\pi; L \rangle
\]

\[
= (-1)^{L + j_\nu + j_\pi + 1} \sqrt{(2L + 1)(2L' + 1)} \left\{ \begin{array}{ccc} L' & j_\nu & j_\pi \\ j_\nu & L & 1 \end{array} \right\}
\]

\[
\times \left[ g_{\ell \nu} \langle j_\nu || j_\nu \rangle + (g_{s \nu} - g_{\ell \nu}) \langle j_\nu || s_\nu \rangle \langle j_\nu \rangle \right],
\]

Table IV. Excitation energies (in MeV), \( B(M1; L \rightarrow L - 1) \) values (in \( \mu^2 \)) and effective angles (in degrees) for two-particle state.

<table>
<thead>
<tr>
<th>( L^\pi )</th>
<th>Excitation energy ( B(M1) )</th>
<th>Effective angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^-)</td>
<td>0.0867</td>
<td>170°</td>
</tr>
<tr>
<td>2(^-)</td>
<td>0.0689</td>
<td>5.1</td>
</tr>
<tr>
<td>3(^-)</td>
<td>0.0446</td>
<td>6.3</td>
</tr>
<tr>
<td>4(^-)</td>
<td>0.0167</td>
<td>6.4</td>
</tr>
<tr>
<td>5(^-)</td>
<td>-0.0111</td>
<td>6.1</td>
</tr>
<tr>
<td>6(^-)</td>
<td>-0.0340</td>
<td>5.6</td>
</tr>
<tr>
<td>7(^-)</td>
<td>-0.0462</td>
<td>4.8</td>
</tr>
<tr>
<td>8(^-)</td>
<td>-0.0411</td>
<td>3.9</td>
</tr>
<tr>
<td>9(^-)</td>
<td>-0.0111</td>
<td>2.7</td>
</tr>
<tr>
<td>10(^-)</td>
<td>0.0524</td>
<td>1.5</td>
</tr>
</tbody>
</table>
and
\[
\langle j_\nu j_\pi; L' \mid (g_{\ell\nu} l_{\pi} + g_{s\nu} s_{\pi}) \mid j_\nu j_\pi; L \rangle = (-1) L' + j_\nu + j_\pi + 1 \sqrt{(2L + 1)(2L' + 1)} \begin{bmatrix} L' & j_\pi & j_\nu \\ j_\pi & L & 1 \end{bmatrix} \\
\times \left[ g_{\ell\nu} \langle j_\pi \mid j_\pi \rangle + (g_{s\nu} - g_{\ell\nu}) \langle j_\pi \mid s_{\pi} \rangle \right]. \quad (A.5)
\]

Here, the reduced matrix elements, \( \langle j \mid j \rangle \) and \( \langle j \mid s \rangle \), are given by
\[
\langle j \mid j \rangle = \sqrt{j(j+1)(2j+1)}, \quad (A.6)
\]
\[
\langle j \mid s \rangle = \frac{1}{2} \sqrt{\frac{2j + 1}{j(j+1)}} \left[ j(j+1) + \frac{3}{4} - \ell^2 \right]. \quad (A.7)
\]

Using the reduced matrix element of the M1 transition operator, we can express the value \( B(M1; L \rightarrow L - 1) \) as
\[
B(M1; L \rightarrow L - 1) = \frac{1}{2L + 1} \left| \langle j_\nu j_\pi; L - 1 \mid T(M1) \mid j_\nu j_\pi; L \rangle \right|^2. \quad (A.8)
\]

The numerical values for \( j_\nu = 11/2, j_\pi = 9/2, g_{\ell\nu} = 0.00, g_{\ell\pi} = 1.00, g_{s\nu} = -2.68, \) and \( g_{s\pi} = 3.91 \) are given in Table IV.

The matrix element of the scalar product of neutron and proton angular momenta is
\[
\langle j_\nu j_\pi; L \mid j_\nu \cdot j_\pi \mid j_\nu j_\pi; L \rangle = \frac{L(L+1) - j_\nu (j_\nu + 1) - j_\pi (j_\pi + 1)}{2}. \quad (A.9)
\]

The matrix element of the squared angular momentum operator of neutrons or protons becomes
\[
\langle j_\nu j_\pi; L \mid j_\tau^2 \mid j_\nu j_\pi; L \rangle = j_\pi (j_\pi + 1). \quad (A.10)
\]
Thus the effective angle is given by
\[
\cos \theta = \frac{L(L+1) - j_\nu (j_\nu + 1) - j_\pi (j_\pi + 1)}{2 \sqrt{j_\nu (j_\nu + 1) j_\pi (j_\pi + 1)}}. \quad (A.11)
\]

The numerical values of the effective angle for \( j_\nu = 11/2 \) and \( j_\pi = 9/2 \) are given in Table IV.

References