Particle Spectra and Gauge Unification in $SU(6) \times SU(2)_R$ Model

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We study the path from the string scale physics to the low-energy physics in the $SU(6) \times SU(2)_R$ string-inspired model with the flavor symmetry $Z_M \times Z_N \times \bar{D}_4$. The flavor symmetry controls the mass spectra of heavy particles as well as those of quarks and leptons in the intermediate energy region ranging from the string scale ($\sim 10^{18}$ GeV) to the electroweak scale. In this paper we examine the mass spectra of heavy particles in detail in our model. The renormalization group evolution of the gauge couplings is studied up to two-loop order. A consistent solution of the gauge unification around the string scale is found by adjusting the spectra of the anti-generation matter fields.

§1. Introduction

The study of the path for connecting the string theory with the low-energy physics is one of the most important issues in particle physics and cosmology. However, we are still in the early stages of the study. In fact, we do not yet have a comprehensive string-based understanding of apparent characteristic patterns in quark and lepton masses and mixings at low energies. It is considered that these characteristic patterns are closely linked to the flavor symmetry, which is expected to arise from the symmetric structure of the compact space in the string theory. It is also likely that the flavor symmetry controls the mass spectra of heavy particles in the intermediate energy region as well as those of quarks and leptons.

Recent developments in the string theory have provided new aspects of string phenomenology. It has been pointed out in Ref. 1) that a new type of non-Abelian flavor symmetry can appear additionally if the compact space is non-commutative. As a matter of fact, in a string with discrete torsion, the coordinates become non-commutative operators.2)–4) As a phenomenological approach, in Refs. 1) and 5) the flavor symmetry $Z_M \times Z_N \times \bar{D}_4$ has been introduced into the string-inspired model, where the cyclic group $Z_M$ and the binary dihedral group $\bar{D}_4$ have R symmetries, while $Z_N$ has not.5) The introduction of a binary representation of non-Abelian flavor symmetry is motivated also by the phenomenological observation that the

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1) We use here the notation “~” for the binary group. In Ref. 5) we have discussed the projective representation of the dihedral group $D_4$, which is identical with the binary dihedral group $\bar{D}_4$. 
R-handed Majorana neutrino mass for the third generation is nearly equal to the geometrical average of the string scale $M_S$ ($\sim 10^{18}$ GeV) and the electroweak scale $M_Z$. In Ref. 5, solving the anomaly-free conditions under many phenomenological constraints coming from the particle spectra, we were led to a large mixing angle (LMA)-MSW solution with $(M, N) = (19, 18)$, in which the appropriate flavor charges are assigned to the matter fields. The solution includes phenomenologically acceptable results concerning fermion masses and mixings and also concerning hierarchical energy scales including the string scale, $\mu$ scale and the Majorana mass scale of R-handed neutrinos.

In the framework of the string theory, we are prohibited from adding extra matter fields by hand. This situation is in sharp contrast to that of the conventional GUT-type models. In fact, in the perturbative heterotic string theory we have no adjoint or higher representation matter (Higgs) fields. Within this rigid framework, we have discussed the path from the string scale physics to the low-energy physics in the $SU(6) \times SU(2)_R$ string-inspired model with the flavor symmetry $Z_{19} \times Z_{18} \times \tilde{D}_4$. In Ref. 6 the renormalization group (RG) equations down from the $M_S$ have been studied for the scalar masses squared of the gauge non-singlet matter fields. It has been found that the radiative breaking of the gauge symmetry can occur slightly below the $M_S$ due to the large Yukawa coupling which is identical with the colored Higgs coupling. This symmetry breaking triggers a subsequent symmetry breaking.\(^6\)

Then, we obtain the sequential symmetry breaking

$$SU(6) \times SU(2)_R \longrightarrow SU(4)_{PS} \times SU(2)_L \times SU(2)_R \longrightarrow G_{SM},$$

where $SU(4)_{PS}$ and $G_{SM}$ represent the Pati-Salam $SU(4)$\(^{10}\) and the standard model gauge group, respectively.

The purpose of this paper is to pursue the further exploration of the path from the string scale physics to the low-energy physics. For this purpose we examine the particle spectra in the intermediate energy region in detail in the $SU(6) \times SU(2)_R$ model with the flavor symmetry $Z_{19} \times Z_{18} \times \tilde{D}_4$. Afterward, the RG runnings of the gauge couplings are studied up to two-loop order. It should be emphasized that in our model the unification scale is not the so-called GUT scale ($\sim 2 \times 10^{16}$ GeV) but around the string scale ($\sim 10^{18}$ GeV). We find a solution of the gauge coupling unification by adjusting the spectra of the anti-generation matter fields, with which the Kähler class moduli fields couple. In this solution $SU(3)_c$ and $SU(2)_L$ gauge couplings meet at $O(5 \times 10^{17}$ GeV). However, $SU(6)$ and $SU(2)_R$ gauge couplings are not perturbatively unified at the string scale. We expect that the non-perturbative unification of $SU(6)$ and $SU(2)_R$ gauge couplings is properly realized in the framework of the higher-dimensional underlying theory.

This paper is organized as follows. In §2, we explain main features of the $SU(6) \times SU(2)_R$ model with the flavor symmetry $Z_{19} \times Z_{18} \times \tilde{D}_4$. Although we have never yet found a concrete example of the string compactification which induces this type of the flavor symmetry exactly, $Z_M \times Z_N$ type of the flavor symmetry appears in some kinds of the Calabi-Yau compactification.\(^7\),\(^8\) Further, the binary type of the flavor symmetry is expected to stem from the compact space with non-commutative geometry. In this paper we assume $Z_{19} \times Z_{18} \times \tilde{D}_4$ as the flavor symmetry. In
§3, we examine the particle spectra in the intermediate energy region ranging from the \( M_S \) to the \( M_Z \). In the intermediate energy region there appear rich spectra of extra heavy particles beyond the minimal supersymmetric standard model. After presenting the two-loop RG equations of the gauge couplings in the intermediate region, we carry out the numerical analysis of the RG runnings of the gauge couplings in §4. We explore solutions of the gauge coupling unification. The final section is devoted to summary and discussion. In Appendix we explain dominant effective Yukawa couplings contributing to the RG evolution of the gauge couplings in the intermediate energy region.

§2. \( SU(6) \times SU(2)_R \) model with the flavor symmetry

Let us start by briefly summarizing the main points of the \( SU(6) \times SU(2)_R \) string-inspired model considered here. More detailed descriptions are given in Refs. 1, 5), 6), 11)-15).

1. The gauge group is assumed to be \( G = SU(6) \times SU(2)_R \), which can be obtained from \( E_6 \) through the \( Z_2 \) flux breaking on a multiply-connected manifold \( K \).16)-18) In contrast to the conventional GUT-type models, we have no Higgs fields of adjoint or higher representations. Nevertheless, the symmetry breaking of \( G \) down to \( G_{SM} \) can take place via the Higgs mechanism.19)

2. In the context of the string theory, it is assumed that the gauge non-singlet matter content consists of the chiral superfields of three families and the single vector-like multiplet in the form

\[
3 \times 27(\Phi_{1,2,3}) + (27(\Phi_0) + \overline{27}(\overline{\Phi}))
\]

in terms of \( E_6 \). The superfields \( \Phi \) in \( 27 \) of \( E_6 \) are decomposed into two irreducible representations of \( G = SU(6) \times SU(2)_R \) as

\[
\Phi(27) = \left\{ \begin{array}{c}
\phi(15, 1) : Q, L, g, g^c, S, \\
\psi(6^*, 2) : (U^c, D^c), (N^c, E^c), (H_u, H_d),
\end{array} \right. \]

where the pair \( g \) and \( g^c \) and the pair \( H_u \) and \( H_d \) represent the colored Higgs and the doublet Higgs superfields, respectively, \( N^c \) is the R-handed neutrino superfield, and \( S \) is an \( SO(10) \) singlet. It should be noted that the doublet Higgs and the color-triplet Higgs fields belong to different irreducible representations of \( G \), as shown in Eq. (2). As a consequence, the triplet-doublet splitting problem is solved naturally.11) In our model there are only two types of gauge invariant trilinear combinations

\[
(\phi(15, 1))^3 = QQg + Qg^cL + g^cggS,
\]

\[
\phi(15, 1)(\psi(6^*, 2))^2 = QH_dD^c + QH_uU^c + LH_dE^c + LH_uN^c \\
+ SH_uH_d + gN^cD^c + gE^cU^c + g^cU^cD^c.
\]
Table I. Assignment of $Z_{342}$ charges for matter superfields. $\theta$ has the charge 18.

<table>
<thead>
<tr>
<th></th>
<th>$\Phi_1$</th>
<th>$\Phi_2$</th>
<th>$\Phi_3$</th>
<th>$\Phi_0$</th>
<th>$\bar{\Phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi(15, 1)$</td>
<td>$a_1 = 126$</td>
<td>$a_2 = 102$</td>
<td>$a_3 = 46$</td>
<td>$a_0 = 12$</td>
<td>$\bar{a} = -16$</td>
</tr>
<tr>
<td>$\psi(6^*, 2)$</td>
<td>$b_1 = 120$</td>
<td>$b_2 = 80$</td>
<td>$b_3 = 16$</td>
<td>$b_0 = -14$</td>
<td>$\bar{b} = -67$</td>
</tr>
</tbody>
</table>

Table II. Assignment of "$\tilde{D}_4$ charges" to matter superfields. The "$\tilde{D}_4$ charge" of $\theta$ is $\sigma_1$.

<table>
<thead>
<tr>
<th></th>
<th>$\Phi_1$ ($i = 1, 2, 3$)</th>
<th>$\Phi_0$</th>
<th>$\bar{\Phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi(15, 1)$</td>
<td>$\sigma_1$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\psi(6^*, 2)$</td>
<td>$\sigma_2$</td>
<td>$\sigma_3$</td>
<td>$\sigma_4$</td>
</tr>
</tbody>
</table>

3. As the flavor symmetry, we introduce the $Z_{19} \times Z_{18}$ and $\tilde{D}_4$ symmetries and regard $Z_{19}$ and $Z_{18}$ as the R and non-R symmetries, respectively. $\tilde{D}_4$ represents the binary dihedral group. Because the numbers 19 and 18 are relatively prime, we can combine these symmetries as

$$Z_{19} \times Z_{18} = Z_{342}. \tag{5}$$

Solving the anomaly-free conditions under the many phenomenological constraints coming from the particle spectra, we obtain a LMA-MSW solution with the $Z_{342}$ charges of the matter superfields, as shown in Table I.\(^5\) In this solution we assign the charge $(-1, 0)$ under $Z_{19} \times Z_{18}$, i.e., the charge 18 under $Z_{342}$ to the Grassmann number $\theta$. The assignment of the "$\tilde{D}_4$ charges" to the matter superfields is given in Table II, where $\sigma_i$ ($i = 1, 2, 3$) represent the Pauli matrices and

$$\sigma_4 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \tag{6}$$

The $\sigma_3$ transformation yields the R-parity. It is found that the R-parities of the superfields $\Phi_i$ ($i = 1, 2, 3$) for the three generations are all odd, while those of $\Phi_0$ and $\bar{\Phi}$ are even.

Due to the gauge symmetry and the flavor symmetry, the superpotential terms which induce the effective Yukawa couplings at low energies take the forms

$$W_Y = \frac{1}{3!} z_0 \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^{\xi_0} (\phi_0)^3 + \frac{1}{3!} z \left( \frac{\phi_0 \bar{\phi}}{M_2^2} \right)^{\xi} (\bar{\phi})^3 + \frac{1}{2} h_0 \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^{\eta_0} \phi_0 \psi_0 \psi_0 \psi_0$$

$$+ \frac{1}{2} \bar{h} \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^{\eta} \left( \frac{\psi_0 \bar{\psi}}{M_2^2} \right)^{\xi} \bar{\psi} \psi \psi + \frac{1}{2} \sum_{i,j=1}^3 z_{i,j} \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^{\xi_{i,j}} \phi_0 \phi_i \phi_j$$

$$+ \frac{1}{2} \sum_{i,j=1}^3 h_{i,j} \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^{\eta_{i,j}} \phi_0 \psi_i \psi_j + \sum_{i,j=1}^3 m_{i,j} \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^{\mu_{i,j}} \psi_0 \phi_i \psi_j, \tag{7}$$

where $\phi_0 \bar{\phi}$ and $\psi_0 \bar{\psi}$ stand for the gauge-singlet combinations. The scale $M_1$ ($M_2$) represents the string scale $M_S$ multiplied by the $O(1)$ factor coming from the volume
of the compact space in which the matter fields \( \phi_0(15, 1) \) and \( \bar{\phi}(15^*, 1) \) \( (\psi_0(6^*, 2) \) and \( \bar{\psi}(6, 2) \) reside. The exponents are determined by the constraints coming from the flavor symmetry and concretely given by

\[
(z_0, \bar{z}, \eta_0, \bar{\eta}) = (0, 150, 158, 84), \quad \zeta_{ij} = \begin{pmatrix} 57 & 51 & 37 \\ 51 & 45 & 31 \\ 37 & 31 & 17 \end{pmatrix},
\]

\[
\eta_{ij} = \begin{pmatrix} 54 & 44 & 28 \\ 44 & 34 & 18 \\ 28 & 18 & 2 \end{pmatrix}, \quad \mu_{ij} = \begin{pmatrix} 49 & 39 & 23 \\ 43 & 33 & 17 \\ 29 & 19 & 3 \end{pmatrix}.
\]

The coefficients \( z_0, \bar{z}, h_0, \bar{h}, z_{ij}, h_{ij} \) and \( m_{ij} \) are \( \mathcal{O}(1) \) constants. The relation \( \zeta_0 = 0 \) means that only the superfield \( \phi_0 \) takes part in the renormalizable interaction with the large Yukawa coupling at \( M_S \). All or some of the powers of the gauge-singlet combination \( \phi_0 \bar{\phi} \) can be replaced with those of another gauge-singlet combination \( \psi_0 \bar{\psi} \) subject to the flavor symmetry.

Here it is important for us to comment on the role of the moduli fields. In the Calabi-Yau string, the generation matter and the anti-generation matter couple separately with the complex structure moduli fields and the Kähler class moduli fields, respectively, in the superpotential.\(^{20}\) The nonvanishing vacuum expectation values (VEVs) of the moduli fields are expected to be \( \mathcal{O}(M_S) \). The VEVs of the Kähler class moduli fields represent the size and shapes of the compact space. In addition, the Kähler class moduli fields carry the flavor charges and their VEVs control the flavor symmetry. Therefore, there is a possibility that the second and the fourth terms in Eq. (7) are supplemented with the other terms multiplied by a certain function of the Kähler class moduli fields. In the next section we discuss the possible modification of the second term.

When \( \phi_0 \) and \( \bar{\phi} \) develop non-zero VEVs, the above non-renormalizable terms induce effective Yukawa couplings with hierarchical patterns. Namely, below the scale \( |\langle \phi_0 \rangle| \), the Froggatt-Nielsen mechanism acts for non-renormalizable interactions in the superpotential.\(^{21}\) Further, we have another non-renormalizable terms

\[
W_1 = M_1^3 \left[ \lambda_0 \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^{2n} + \lambda_1 \left( \frac{\phi_0 \bar{\phi}}{M_1^2} \right)^n \left( \frac{\psi_0 \bar{\psi}}{M_2^2} \right)^m + \lambda_2 \left( \frac{\psi_0 \bar{\psi}}{M_2^2} \right)^{2m} \right]
\]

with \( \lambda_i = \mathcal{O}(1) \). The flavor symmetry \( Z_{342} \times \tilde{D}_4 \) requires \( n = 81 \) and \( m = 4 \). We assume that the supersymmetry is broken at the string scale due to the hidden sector dynamics and that the supersymmetry (SUSY) breaking is communicated gravitationally to the observable sector via the universal soft SUSY breaking terms. The scale of the SUSY breaking \( m_\phi \) is supposed to be \( 10^3 \) GeV. Under this assumption we study the minimum point of the scalar potential.\(^9\) When we have a large coupling for the Yukawa interaction \( z_0 \phi_0^3 \) at the string scale \( M_S \), through the RG evolution the scalar masses squared become negative slightly below \( M_S \).\(^6\) As a consequence, the ground state is characterized by \( \langle \phi_i \rangle = \langle \psi_i \rangle = 0, \,(i = 1, 2, 3) \) and

\[
\frac{|\langle \phi_0 \rangle|}{M_1} = \frac{|\langle \bar{\phi} \rangle|}{M_1} = 0.894, \quad \frac{|\langle \psi_0 \rangle|}{M_2} = \frac{|\langle \bar{\psi} \rangle|}{M_2} = 0.103,
\]

\( \frac{1}{M_1} \).
where we take the numerical values $M_1 = M_2 = 5 \times 10^{17}$ GeV $= M_S/3$ and

$$
\left( \frac{|\langle \phi_0 \rangle|}{M_1} \right)^{4n-2} \times M_1 = c \tilde{m}_0.
$$

(11)

The coefficient $c$ on the r.h.s. is expressed as a function of $\lambda_{0,1,2}$, $n$ and $m$ and we take here $c = 0.1$. In this vacuum we have the relations

$$
x^{81} \simeq y^4, \quad x^{161} M_1 = 10^2 \text{ GeV},
$$

(12)

where we use the notation $x = (|\langle \phi_0 \rangle|/M_1)^2$ and $y = (|\langle \psi_0 \rangle|/M_2)^2$. Thus, below the scale $|\langle \psi_0 \rangle|$ we obtain the effective Yukawa superpotential

$$
W^{(\text{eff})}_Y = \frac{1}{3!} z_0 x\phi_0 (\phi_0)^3 + \frac{1}{3!} \bar{z} \bar{x}(\bar{\phi}) \bar{\phi}^3 + \frac{1}{2} h_0 x^\eta \phi_0 \psi_0 \psi_0
$$

$$
+ \frac{1}{2} \bar{h} \bar{x} y^2 \bar{\phi} \psi \bar{\psi} + \frac{1}{2} \sum_{i,j=1}^3 z_{ij} x^{\xi_{ij}} \phi_0 \phi_i \phi_j
$$

$$
+ \frac{1}{2} \sum_{i,j=1}^3 h_{ij} x^{\eta_{ij}} \phi_0 \psi_i \psi_j + \sum_{i,j=1}^3 m_{ij} x^{\mu_{ij}} \psi_0 \phi_i \psi_j.
$$

(13)

§3. Mass spectra of heavy particles

In this section we explore the particle spectra in the intermediate energy region ranging from the $M_S$ to the $M_Z$ for the R-parity even superfields first and then for the odd superfields.

A. The R-parity even superfields

The R-parity even superfields contain $\phi_0$, $\bar{\phi}$, $\psi_0$ and $\bar{\psi}$. As mentioned above, the gauge symmetry is spontaneously broken at the scale $|\langle \phi_0(15, 1) \rangle| \simeq 4.5 \times 10^{17}$ GeV, and subsequently at the scale $|\langle \psi_0(6*, 2) \rangle| \simeq 5.2 \times 10^{16}$ GeV. This yields the symmetry breakings

$$
SU(6) \times SU(2)_R \xrightarrow{\langle \phi_0 \rangle} SU(4)_{PS} \times SU(2)_L \times SU(2)_R \xrightarrow{\langle \psi_0 \rangle} G_{SM}.
$$

(14)

Since the fields that develop non-zero VEVs are singlets under the remaining gauge symmetries, they are assigned as $\langle \phi_0(15, 1) \rangle = \langle S_0 \rangle$ and $\langle \psi_0(6*, 2) \rangle = \langle N_0^c \rangle$. In the first step of the symmetry breaking, the fields $Q_0$, $L_0$, $\bar{Q}$, $\bar{L}$ and $(S_0 - S)/\sqrt{2}$ are absorbed by the gauge fields. Through the subsequent symmetry breaking, the fields $U_0^c$, $E_0^c$, $\bar{U}^c$, $\bar{E}^c$ and $(N_0^c - N^c)/\sqrt{2}$ are absorbed. Therefore, below the scale $|\langle \psi_0 \rangle|$ the remaining modes in the R-parity even superfields are gauge singlet fields.
(\(S' \equiv (S_0 + \overline{S})/\sqrt{2}\), \(N'^c \equiv (N_0^c + \overline{N}^c)/\sqrt{2}\)), doublet Higgs fields \((H_{u0}, H_{d0}, \overline{H}_u, \overline{H}_d)\) and down-type colored fields \((g_0, g_0^c, D_0^c, \overline{g}, \overline{g}^c, \overline{D}^c)\).

1. Gauge singlet fields \(S'\) and \(N'^c\)
Mass matrix for \(S'\) and \(N'^c\) is induced from Eq. (9) as

\[
\tilde{M}_{S', N'^c} = \begin{pmatrix}
\begin{array}{cc}
S' \\
N'^c
\end{array}
\end{pmatrix}
\begin{pmatrix}
\lambda_0 \mathcal{O}(4n^2)x^{2n-1} & \lambda_1 \mathcal{O}(nm)x^{2n(1-1/4m)-1/2} \\
\lambda_1 \mathcal{O}(nm)x^{2n(1-1/4m)-1/2} & \lambda_2 \mathcal{O}(4m^2)x^{2n(1-1/2m)}
\end{pmatrix},
\]

in \(M_1\) units, where \(n = 81\) and \(m = 4\). The eigenvalues are given by

\[
\mathcal{O}(x^{118} M_1) \sim 10^{6.2} \text{ GeV}, \quad \mathcal{O}(x^{125} M_1) \sim 10^{5.5} \text{ GeV}.
\]

In the following we use the notation \(\tilde{S}\) and \(\tilde{N}^c\) for the mass eigenstates.

2. Doublet Higgs fields \(H_{u0}, H_{d0}, \overline{H}_u\) and \(\overline{H}_d\)
The mass matrix for these fields is derived from the terms in Eqs. (9) and (13). The effective superpotential contributing to this mass matrix becomes

\[
W_H^{(\text{eff})} \simeq h_0 x^{\eta_0} S_0 H_{u0} H_{d0} + \overline{h} x^{\overline{\eta}} y^2 \overline{S} H_u \overline{H}_d + \lambda_1 x^n y^{m-1} (H_{u0} \overline{H}_u + H_{d0} \overline{H}_d),
\]

where the first term induces the so-called \(\mu\)-term with \(\mu = \mathcal{O}(x^{\eta_0+1/2} M_1)\). Using \((\eta_0, \overline{\eta}) = (158, 84)\) and \(x^{81} \simeq y^4\), we obtain the mass matrix

\[
\tilde{M}_H \simeq \begin{pmatrix}
H_{u0} \\
H_{d0} \\
\overline{H}_u \\
\overline{H}_d
\end{pmatrix}
\begin{pmatrix}
H_{d0} \\
\lambda_0 x^{158.5} \\
\lambda_1 x^{141.75} \\
\overline{h} x^{125}
\end{pmatrix},
\]

in \(M_1\) units. The eigenvalues are

\[
\mathcal{O}(x^{125} M_1) \sim 10^{5.5} \text{ GeV}, \quad \mathcal{O}(x^{158.5} M_1) \sim 10^{2.2} \text{ GeV}.
\]

3. Down-type colored fields \(g_0, g_0^c, D_0^c, \overline{g}, \overline{g}^c\) and \(\overline{D}^c\)
The effective superpotential which yields the mass matrix for these fields is of the form

\[
W_g^{(\text{eff})} \simeq z_0 x^{\zeta_0} S_0 g_0 g_0^c + \overline{z} x^{\overline{\zeta}} \overline{S} \overline{g} \overline{g}^c + h_0 x^{\eta_0} g_0 N_0^c D_0^c + \overline{h} x^{\overline{\eta}} \overline{g} \overline{N}^c \overline{D}^c + \lambda_0 x^{2n-1} (g_0 \overline{g} + g_0 \overline{g}^c) + \lambda_1 x^n y^{m-1} D_0^c \overline{D}^c.
\]

This effective superpotential leads to the mass matrix

\[
\tilde{M}_g \simeq \begin{pmatrix}
g_0 \\
\overline{g} \\
D^c
\end{pmatrix}
\begin{pmatrix}
\lambda_0 x^{0.5} & \lambda_0 x^{161} & h_0 x^{168.125} \\
\lambda_0 x^{161} & \overline{z} x^{150.5} & 0 \\
0 & \overline{h} x^{134.625} & \lambda_1 x^{141.75}
\end{pmatrix}.
\]
in $M_1$ units. Here, $(2, 3)$ and $(3, 1)$ elements in this matrix are not exactly zero but approximately zero. In fact, the terms $\bar{S}N^c_0\bar{g}^cD_0^c$ and $S_0N^c_0g_0^cD^c$ are induced through the higher order effects and sufficiently suppressed compared to the terms in Eq. (20). The eigenvalues of this matrix are $O(x^{0.5}M_1)$, $O(x^{134.625}M_1)$ and $O(x^{157.625}M_1)$. So it turns out that one set of down-type colored superfields with even $R$-parity should exist around $x^{157.625}M_1 \approx O(10^{2.3} \text{ GeV})$. As pointed out in the previous section, however, in the Calabi-Yau string the anti-generation matter fields couple with the Kähler class moduli fields, which carries the flavor charge. Therefore, both the second and the fourth terms in Eq. (13), i.e., $\bar{\phi}^3$ and $\bar{\phi}\bar{\psi}^2$ terms, are possibly supplemented with the other terms. Since we have never yet known the definite content of the Kähler class moduli fields and also their flavor charges, the explicit form of the interactions of the Kähler class moduli fields is undetermined in the present approach. In this paper, from the phenomenological viewpoint, we consider a simple case that one of the Kähler class moduli fields couple with the anti-generation fields $\bar{\phi}$ only via the interaction

$$f(T) \left( \frac{S_0\bar{S}}{M_1^2} \right)^{\bar{k}} \phi^3, \quad (22)$$

where $T$ is one of the Kähler class moduli fields and carries an appropriate flavor charge. The exponent $\bar{k}$ is introduced as an unknown parameter, because we cannot determine the flavor charge of $T$ at this stage as well as the explicit form of the function $f(T)$. In view of the fact that the Kähler class moduli fields represent the size and the shape of the compact space, it is supposed that the field $T$ develops a non-zero VEV with $\langle T \rangle = O(M_S)$ and $f(\langle T \rangle) = O(1)$. If $\bar{k} < 150.5$, a dominant term contributing to the $(2, 2)$ element in Eq. (21) is replaced by

$$f(\langle T \rangle) x^k \langle \bar{S} \rangle \bar{g}\bar{g}^c = M_1 f(\langle T \rangle) x^k \bar{g}\bar{g}^c, \quad (23)$$

where $k = \bar{k} + 0.5$. Then the mass matrix is also replaced by

$$\mathbf{\hat{M}}_g \simeq \frac{g_0}{\bar{g}^c} \frac{D^c}{D_0^c} \begin{pmatrix} g_0^c & \bar{g} & D_0^c \\ z_0 x^{0.5} & \lambda_0 x^{161} & h_0 x^{168.125} \\ \lambda_0 x^{161} & f(\langle T \rangle) x^k & 0 \\ 0 & \bar{h} x^{134.625} & \lambda_1 x^{141.75} \end{pmatrix} \quad (24)$$

in $M_1$ units. If we take $10.125 < k < 134.625$, the eigenvalues of this matrix become

$$O(x^{0.5}M_1) \sim 10^{17.7} \text{ GeV}, \quad O(x^kM_1), \quad O(x^{141.75}M_1) \sim 10^{3.9} \text{ GeV} \quad (25)$$

with $x^{10.125}M_1 \sim 10^{16.7} \text{ GeV} > x^kM_1 > x^{134.625}M_1 \sim 10^{4.6} \text{ GeV}$. In the next section we adjust the parameter $k$ so as to find a consistent solution of gauge unification. Thus, the Kähler class moduli fields, although are gauge-singlets, possibly affect the evolution of the gauge couplings through the couplings with the anti-generation fields.
Table III. The multiplicities and the spectra of the R-parity even superfields. In this table the parameter $k$ is supposed to be $10.125 < k < 118$, because $k$ is adjusted as $k = 42.5$ later.

<table>
<thead>
<tr>
<th>$\times M_1$</th>
<th>$Q$</th>
<th>$L$</th>
<th>$U^c$</th>
<th>$E^c$</th>
<th>$(H_u, H_d)$</th>
<th>$(g, g^c, D^c)$</th>
<th>$(S, N^c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$\sim$</td>
<td>$x^{0.5}$</td>
<td>$2$</td>
<td>$2$</td>
<td>$2$</td>
<td>$4$</td>
<td>$6$</td>
</tr>
<tr>
<td>$x^{0.5}$</td>
<td>$\sim$</td>
<td>$x^{10.125}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$2$</td>
<td>$2$</td>
<td>$4$</td>
</tr>
<tr>
<td>$x^{10.125}$</td>
<td>$\sim$</td>
<td>$x^k$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$4$</td>
<td>$4$</td>
</tr>
<tr>
<td>$x^k$</td>
<td>$\sim$</td>
<td>$x^{118}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$4$</td>
<td>$2$</td>
</tr>
<tr>
<td>$x^{118}$</td>
<td>$\sim$</td>
<td>$x^{125}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$4$</td>
<td>$2$</td>
</tr>
<tr>
<td>$x^{125}$</td>
<td>$\sim$</td>
<td>$x^{141.75}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$2$</td>
<td>$2$</td>
</tr>
<tr>
<td>$x^{141.75}$</td>
<td>$\sim$</td>
<td>$x^{158.5}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$2$</td>
<td>$0$</td>
</tr>
<tr>
<td>$x^{158.5}$</td>
<td>$\sim$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

We summarize the multiplicities and the spectra of the R-parity even superfields in Table III. We suppose here $10.125 < k < 118$, because the parameter $k$ is adjusted as $k = 42.5$ later.

B. The R-parity odd superfields

The R-parity odd superfields contains three generations of matter superfields $\phi_i$ and $\psi_i$ ($i = 1, 2, 3$). We study here the mass spectra of these fields in order.

1. Up-type quarks
   As seen from the last term in Eq. (13) which contains $m_{ij} x^{\mu_\omega} Q_i U^c_j H_{u0}$, the mass matrix for up-type quarks becomes
   \begin{equation}
   M_{ij} v_u = m_{ij} x^{\mu_{ij}} v_u ,
   \end{equation}
   where $v_u = (H_{u0})$ and $m_{ij} = O(1)$. The exponents $\mu_{ij}$ are given in Eq. (8). The eigenvalues of the mass matrix are
   \begin{equation}
   O(x^{49} v_u), \quad O(x^{33} v_u), \quad O(x^3 v_u),
   \end{equation}
   which correspond to $m_u$, $m_c$ and $m_t$, respectively. Since $v_u = O(10^2 \text{GeV})$, all of up-type quarks remain massless in the intermediate energy region.

2. Down-type colored fields
   The effective superpotential of down-type colored fields with odd R-parity takes the form
   \begin{equation}
   W_D^{(\text{eff})} = \sum_{i,j=1}^3 \left( z_{ij} x^{\xi_{ij}} S_0 g_i g^c_j + m_{ij} x^{\mu_{ij}} N_0^c g_i D^c_j + m_{ij} x^{\mu_{ij}} H_{d0} Q_i D^c_j \right).
   \end{equation}
   When $S_0$, $N_0^c$ and $H_{d0}$ develop nonvanishing VEVs, the mass matrix is derived
as
\[
\hat{\mathcal{M}}_d = \begin{pmatrix} g^c & D^c \\ 0 & 0 \end{pmatrix}
\begin{pmatrix} x^{0.5} & y^{0.5} \mathcal{M} \\ \rho_d \mathcal{M} \end{pmatrix}
\]
(29)
in $M_1$ units. The submatrices $Z$ and $\mathcal{M}$ are given by
\[
Z_{ij} = z_{ij} x^{c_{ij}}
\]
(30)
and Eq. (26), respectively and $\rho_d = v_d / M_1$ with $v_d = \langle H_d \rangle$. The mass matrix $\hat{\mathcal{M}}_d$ yields mixings between $g^c$ and $D^c$ and has six eigenvalues. Three of them represent light modes. Solving the eigenvalue problem of this matrix, we obtain their masses\(^{1,11,14}\)
\[
\mathcal{O}(x^{49} v_d), \quad \mathcal{O}(x^{41} v_d), \quad \mathcal{O}(x^{21} v_d),
\]
(31)
which correspond to $m_d$, $m_s$ and $m_b$, respectively. The remaining three represent heavy modes with their masses
\[
\mathcal{O}(x^{13.125} M_1), \quad \mathcal{O}(x^{31.5} M_1), \quad \mathcal{O}(x^{49.125} M_1).
\]
(32)

3. Charged leptons and extra charged Higgs fields with odd R-parity
For these fields the effective superpotential is written as
\[
W_E^{\text{(eff)}} = \sum_{i,j=1}^{3} (h_{ij} x^{n_{ij}} S_0 H_d H_u + m_{ij} x^{n_{ij}} N_0^c L H_u + m_{ij} x^{n_{ij}} H_d H_0 L E^c_j).
\]
(33)
The mass matrix is similar to that for the down-type colored fields. Concretely, we have
\[
\hat{\mathcal{M}}_l = \begin{pmatrix} H_u & E^c \\ L & 0 \end{pmatrix}
\begin{pmatrix} x^{0.5} \mathcal{H} & 0 \\ y^{0.5} \mathcal{M} & \rho_d \mathcal{M} \end{pmatrix}
\]
(34)
in $M_1$ units, where the submatrix $\mathcal{H}$ is given by
\[
\mathcal{H}_{ij} = h_{ij} x^{n_{ij}}.
\]
(35)
The mass matrix $\hat{\mathcal{M}}_l$ yields $L$-$H_d$ mixings. Among six eigenvalues of $\hat{\mathcal{M}}_l$, three represent light modes corresponding to charged leptons. Their masses are given by\(^{1,11,15}\)
\[
\mathcal{O}(x^{49} v_d), \quad \mathcal{O}(x^{35} v_d), \quad \mathcal{O}(x^{17} v_d),
\]
(36)
which represent $m_e$, $m_\mu$ and $m_\tau$, respectively. The remaining three represent heavy modes with their masses
\[
\mathcal{O}(x^{2.5} M_1), \quad \mathcal{O}(x^{29.125} M_1), \quad \mathcal{O}(x^{44.5} M_1).
\]
(37)
4. Neutral fields with odd R-parity

The neutral sector contains five types of matter fields, \( H_u^0, H_d^0, L^0, N^c \) and \( S \) in our model. Then we have the \( 15 \times 15 \) mass matrix\(^1,11-13,15\)

\[
\tilde{\mathcal{M}}_{NS} = \begin{pmatrix}
H_u^0 & H_d^0 & L^0 & N^c & S \\
\begin{pmatrix}
0 \\
x^{0.5} \mathcal{H} \\
x^{0.5} \mathcal{H} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} & \begin{pmatrix}
0 \\
y^{0.5} \mathcal{M}^T \\
y^{0.5} \mathcal{M}^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} & \begin{pmatrix}
0 \\
0 \\
0 \\
\rho_d \mathcal{M}_T \\
\rho_d \mathcal{M}_T \\
\rho_u \mathcal{M}_T \\
\rho_u \mathcal{M}_T \\
\rho_u \mathcal{M}_T \\
\rho_u \mathcal{M}_T \\
\rho_u \mathcal{M}_T \\
\rho_u \mathcal{M}_T \\
\rho_u \mathcal{M}_T
\end{pmatrix} & \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\end{pmatrix}
\]  

(38)

in \( M_S \) units, where \( \rho_u = v_u/M_1 \). In this mass matrix, the \( 3 \times 3 \) submatrix \( \mathcal{N} \) plays the role of the R-handed Majorana mass matrix in the seesaw mechanism. Among fifteen eigenvalues of \( \tilde{\mathcal{M}}_{NS} \) three represent tiny masses of neutrinos\(^1,15\) which are given by

\[
\frac{v_u^2}{M_1 x^{37.25}} \times (\mathcal{O}(x^{12}), \mathcal{O}(x^4), \mathcal{O}(1)).
\]  

(39)

Six of them are degenerate with the above-mentioned three heavy modes coming from extra charged Higgs fields. The remaining six eigenvalues are approximately given by the submatrices \( \mathcal{N} \) and \( \mathcal{S} \). \( \mathcal{N} \) is induced from the term

\[
\frac{n_{ij}}{M_1} x^{\nu_{ij}} (N_i^c \bar{N}^c)(N_j^c \bar{N}^c)
\]  

(40)

and of the form

\[
\mathcal{N}_{ij} = n_{ij} x^{\nu_{ij}} y
\]  

(41)

in \( M_1 \) units, where the flavor symmetry leads to the relation \( \nu_{ij} = \eta_{ij} + 49 \). The three eigenvalues of \( \mathcal{N} \) become

\[
\mathcal{O}(x^{123.25} M_1), \quad \mathcal{O}(x^{103.25} M_1), \quad \mathcal{O}(x^{71.25} M_1),
\]  

(42)

which represent the masses of the R-handed Majorana neutrinos. The largest eigenvalue \( \mathcal{O}(x^{71.25} M_1) \simeq 10^{10.8} \) GeV is nearly equal to the geometrical average of \( M_S \) and \( M_Z \). The submatrix \( \mathcal{S} \) induced from

\[
\frac{s_{ij}}{M_1} x^{\sigma_{ij}} (S_i \bar{S})(S_j \bar{S})
\]  

(43)

is given by

\[
S_{ij} = s_{ij} x^{\sigma_{ij}+1}
\]  

(44)

in \( M_1 \) units, where we have the relation \( \sigma_{ij} = \zeta_{ij} - 11 \). The eigenvalues of \( \mathcal{S} \) become

\[
\mathcal{O}(x^{47} M_1), \quad \mathcal{O}(x^{35} M_1), \quad \mathcal{O}(x^7 M_1).
\]  

(45)
Table IV. The multiplicities and the spectra of the R-parity odd superfields.

<table>
<thead>
<tr>
<th>$\times M_1$</th>
<th>$Q$</th>
<th>$U^c$</th>
<th>$g$</th>
<th>$g^c$</th>
<th>$E^c$</th>
<th>$H_u$</th>
<th>$(L, H_d)$</th>
<th>$N^c$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$x^{2.5}$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$x^7$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{13.125}$</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{29.125}$</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{31.5}$</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{35}$</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{44.5}$</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{47}$</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{49.125}$</td>
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<td>3</td>
<td>1</td>
<td>4</td>
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<td>2</td>
</tr>
<tr>
<td>$x^{71.25}$</td>
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<td>3</td>
<td>0</td>
<td>3</td>
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<td>2</td>
</tr>
<tr>
<td>$x^{103.25}$</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>3</td>
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<td>0</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{123.25}$</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

In Table IV we summarize the multiplicities of the R-parity odd superfields. It is worthy to note that in the intermediate energy region we have the hierarchical spectra of heavy particles including the R-handed Majorana neutrinos.

§4. RG equations and numerical study

Now we study the two-loop RG evolutions of the gauge couplings in our model. The evolution equations up to the two-loop order for $\alpha_i = g_i^2/4\pi$ are generally given by

$$
\frac{d\alpha_i}{dt} = \frac{1}{2\pi} \left[ -b_i + \frac{1}{4\pi} \left( \sum_j b_{ij} \alpha_j - a_i \right) \right] \alpha_i^2,
$$

(46)

where $t = \ln(Q/Q_0)$ with $Q_0 = M_S$. The subscripts $i$ and $j$ specify the gauge group. The coefficients $b_i$, $b_{ij}$ and $a_i$ are determined depending on the particle contents and their spectra. The $a_i$ terms represent the contributions of Yukawa interactions.

In our model, the values of the gauge couplings $g_6$ of $SU(6)$ and $g_{2R}$ of $SU(2)_R$ are introduced at the string scale $M_S$ as an initial condition. As mentioned above, the gauge group $SU(6) \times SU(2)_R$ is broken down to $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ at the energy scale $|\langle \phi_0 \rangle| = x^{0.5} M_1$. The subsequent breaking of the gauge group into $SU(3)_c \times SU(2)_L \times U(1)_Y$ occurs at the energy scale $|\langle \psi_0 \rangle| = y^{0.5} M_1 \simeq x^{10.125} M_1$. The supersymmetry is supposed to be broken at the scale $m_0 = 10^3 \text{GeV} = x^{151} M_1$. From the particle spectra given in Tables III and IV, we can determine the coefficients $b_i$ and $b_{ij}$ in various energy regions ranging from $M_S$ to $M_Z$.

In the first region between $M_S (t = 0)$ and $x^{0.5} M_1 (t = -1.21)$, where the gauge
with $y_0 = |z_0|^2/4\pi$. As seen from Eqs. (7) and (8), in this region only $z_0(\phi_0)^3$ term contributes to the RG equations for gauge couplings at two-loop level. The subscripts $i, j = 6$ and $2R$ denote $SU(6)$ and $SU(2)_R$, respectively. The one-loop RG equation for the Yukawa coupling $y_0$ is given by

$$\frac{dy_0}{dt} = \frac{1}{2\pi} (-28 \alpha_6 + 9 y_0) y_0.$$  \hfill (48)

As will be discussed later, the evolution of the gauge couplings is rather insensitive to the Yukawa couplings in the whole region. Then, in the present analysis it is sufficient for us to take the one-loop RG equations for the Yukawa couplings into account.

In the second region between $x^{0.5} M_1 (t = -1.21)$ and $x^{10.125} M_1 (t = -3.37)$, the gauge group is $SU(4)_P \times SU(2)_L \times SU(2)_R$. In this region we obtain

$$-b_i = \begin{pmatrix} 0 \\ n_H \\ 4 + n_H \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 100 & 9 & 15 \\ 45 & 18 + 7 n_H & 3 n_H \\ 75 & 3 n_H & 46 + 7 n_H \end{pmatrix},$$

$$a_i = \begin{pmatrix} 4 (y^{(Z)} + 2 y^{(1)} + 2 y^{(2)} + 4 y^{(3)}) \\ 4 (y^{(H)} + 4 y^{(1)} + 4 y^{(2)} + y^{(4)}) \\ 4 (y^{(H)} + 4 y^{(1)} + 4 y^{(2)} + 3 y^{(3)} + y^{(4)}) \end{pmatrix},$$

where $i, j = 4, 2L$ and $2R$ denotes $SU(4)_P$, $SU(2)_L$ and $SU(2)_R$, respectively. The notation $n_H$ in Eq. (49) represents the multiplicity of doublet Higgs fields and is given by

$$n_H = \begin{cases} 5 & x^{0.5} > Q/M_1 \geq x^{2.5}, \\ 4 & x^{2.5} > Q/M_1 \geq x^{10.125}. \end{cases}$$  \hfill (50)

At $x^{0.5} M_1$ we use the continuity condition

$$\alpha_6 = \alpha_4 = \alpha_{2L}.$$  \hfill (51)

The dominant contributions of the effective Yukawa interactions are of the forms

$$y^{(Z)} = \frac{1}{4\pi} |f^{(Z)}_{33}|^2, \quad y^{(1)} = \frac{1}{4\pi} |M^{(1)}_{33}|^2, \quad y^{(3)} = \frac{1}{4\pi} |M^{(3)}_{33}|^2,$$

$$y^{(H)} = \frac{1}{4\pi} |f^{(H)}_{33}|^2 \theta \left( \frac{Q}{M_1} - x^{2.5} \right), \quad y^{(2)} = \frac{1}{4\pi} |M^{(2)}_{33}|^2 \theta \left( \frac{Q}{M_1} - x^{2.5} \right),$$

$$y^{(4)} = \frac{1}{4\pi} |M^{(4)}_{33}|^2 \theta \left( \frac{Q}{M_1} - x^{2.5} \right).$$  \hfill (52)

The definition of the effective Yukawa couplings $f^{(Z,H)}_{33}$ and $M^{(1\sim4)}_{33}$ are presented in the Appendix. The RG equations for the Yukawa couplings are also given in the Appendix.
In the third region ranging from \( x^{10.125} M_1 \) \((t = -3.37)\) to \( x^{151} M_1(= m_\phi = 10^3 \text{GeV}) \) \((t = -35.00)\), where the gauge group coincides with \( G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \), still being supersymmetric, we obtain

\[
-b_i = \begin{pmatrix}
-3 + N_g \\
6 + \frac{2}{5} N_g + \frac{3}{5} n_H \\
14/5
\end{pmatrix} n_H,
\]

\[
b_{ij} = \begin{pmatrix}
18/5 & 11/5 \\
24 & 18 & 6/5 \\
88/5 & 18/5 & 38/5
\end{pmatrix} + \begin{pmatrix}
34/3 & 0 & 4/15 \\
0 & 0 & 0 \\
32/15 & 0 & 8/75
\end{pmatrix} N_g
\]

\[
+ \begin{pmatrix}
0 & 0 & 0 \\
0 & 7 & 3/5 \\
0 & 9/5 & 9/25
\end{pmatrix} n_H,
\]

\[
a_i = \begin{pmatrix}
4 \\
6 \\
26/5
\end{pmatrix} y^{(1U)} + \begin{pmatrix}
4 \\
6 \\
14/5
\end{pmatrix} y^{(1D)}
\]

\[
+ \begin{pmatrix}
1 \\
0 \\
2/5
\end{pmatrix} \left( y^{(3N)} + 2y^{(3D)} + 2y'^{(Z)} \right),
\]

(53)

where \( i, j = 3, 2L \) and 1 means \( SU(3)_c, SU(2)_L \) and \( U(1)_Y \), respectively. \( n_H \) and \( N_g \), the latter being the multiplicity of the extra down-type colored fields, are shown in Table V. The Yukawa terms are expressed as

\[
y^{(1U)} = \frac{1}{4\pi} |M^{(1U)}_{33}|^2,
\]

\[
y^{(1D)} = \frac{1}{4\pi} |M^{(1D)}_{33}|^2 \theta \left( \frac{Q}{M_1} - x^{13.125} \right),
\]

\[
y^{(3N)} = \frac{1}{4\pi} |M^{(3N)}_{33}|^2 \theta \left( \frac{Q}{M_1} - x^{13.125} \right),
\]

\[
y^{(3D)} = \frac{1}{4\pi} |M^{(3D)}_{33}|^2 \theta \left( \frac{Q}{M_1} - x^{13.125} \right),
\]

\[
y'(Z) = \frac{1}{4\pi} |f'_{33}(Z)|^2 \theta \left( \frac{Q}{M_1} - x^{13.125} \right).
\]

(54)

The continuity conditions at the scale \( x^{10.125} M_1 \) are given by

\[
\alpha_4 = \alpha_3,
\]

(55)

\[
\alpha^{-1} = \frac{2}{5} \alpha_4^{-1} + \frac{3}{5} \alpha_2^{-1},
\]

(56)

\[
y^{(1)} = y^{(1U)} = y^{(1D)},
\]

(57)

\[
y^{(3)} = y^{(3N)} = y^{(3D)},
\]

(58)

\[
f'_{33} = x^{4.375} f_{33} + \frac{1}{\sqrt{2}} M^{(3)}_{33}.
\]

(59)

The last condition is due to the mixing of \( g^c \) and \( D^c \) as seen in Eq. (29). In the Appendix we list the definitions of \( f^{(Z)}_{33} \) and \( M^{(1U-3D)}_{33} \) and the RG equations for the Yukawa couplings.
Table V. The values of $n_H$ and $N_g$ in the region between $x^{10.125} M_1$ and $x^{151} M_1$. $n_H$ and $N_g$ are the multiplicities of doublet Higgses and extra down-type colored fields, respectively. Here, the parameter $k$ is taken as $31.5 < k < 44.5$. In the numerical calculation, we use the adjusted value $k = 42.5$.

<table>
<thead>
<tr>
<th>$\times M_1$</th>
<th>$n_H$</th>
<th>$N_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^{10.125}$</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$x^{13.125}$</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$x^{29.125}$</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$x^{31.5}$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$x^k$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$x^{44.5}$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$x^{49.125}$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$x^{125}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x^{141.75}$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

The final region with the gauge symmetry $G_{SM}$ is between $x^{151} M_1 (t = -35.00)$ and $x^{161.4} M_1 (= M_Z) (t = -37.34)$, where the supersymmetry is broken. In this region, we have

$$-b_i = \begin{pmatrix} -7 \\ -\frac{10}{3} + n_H \\ 4 + \frac{3}{5} n_H \end{pmatrix},$$

$$b_{ij} = \begin{pmatrix} -26 & 9/2 & 11/10 \\ 12 & 11/3 & 3/5 \\ 44/5 & 9/5 & 19/5 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 25/2 & 9/10 \\ 0 & 27/10 & 27/50 \end{pmatrix} n_H,$$

$$a_i = \begin{pmatrix} 2 \\ 3/2 \\ 17/10 \end{pmatrix} y^{(1U)},$$

where

$$n_H = \begin{cases} 1 & x^{151} > Q/M_1 \geq x^{158}, \\ 0 & x^{158} > Q/M_1 \geq x^{161.4}. \end{cases}$$

The evolution equation for the Yukawa coupling is given by

$$\frac{dy^{(1U)}}{dt} = \frac{1}{2\pi} \left[ -8\alpha_3 - \frac{9}{4}\alpha_2 L - \frac{17}{20} \alpha_1 + \frac{9}{2} y^{(1U)} \right] y^{(1U)}.$$

We are now in a position to solve the RG equations numerically. By adjusting the relevant parameters, we can obtain some consistent solutions of the gauge unification. A typical solution is shown in Fig. 1, where the initial values of gauge coupling constants at the string scale $M_S$ are taken as

$$\alpha_6 (t = 0) = 0.057, \quad \alpha_2 R (t = 0) = 0.083$$

and that of the Yukawa coupling as $y_0 (t = 0) = 0.6$. In this solution the values of $z_{33}, h_{33}$ and $m_{33}$ in Eq. (13) are taken as 1.0, 0.3 and 2.0, respectively, and the
parameter $k$ is adjusted as $k = 42.5$. Under this choice of the parameter values, we obtain the running coupling constants at $M_Z$ as

$$\alpha_3^{-1}(M_Z) = 8.44, \tag{64}$$

$$\alpha_2^{-1}(M_Z) = 29.79, \tag{65}$$

$$\alpha_1^{-1}(M_Z) = 59.48. \tag{66}$$

These results are in good agreement with the experimental values$^{23}$). Numerically, the results are insensitive to the values of $y_{33}$, $h_{33}$ and $m_{33}$ but appreciably affected by the value of $k$. For instance, if we take $k = 37.5$ instead of $k = 42.5$, the coupling constants are changed as

$$\alpha_3^{-1}(M_Z) = 8.26, \tag{67}$$

$$\alpha_2^{-1}(M_Z) = 29.79, \tag{68}$$

$$\alpha_1^{-1}(M_Z) = 59.41. \tag{69}$$

In our model the spectra of the anti-generation fields $\bar{g}$ and $\bar{g}^c$ play an important role in adjusting the gauge couplings. Since the fields $\bar{g}$ and $\bar{g}^c$ are colored but $SU(2)_L$-singlets, $\alpha_3(M_Z)$ and $\alpha_1(M_Z)$ increase with decreasing $k$ but $\alpha_{2L}(M_Z)$ is almost independent of $k$. We find a consistent solution of the gauge coupling unification, which represents the connection between the string scale physics and the electroweak scale physics.
§5. Summary and discussion

Characteristic patterns in fermion masses and mixings at low energies strongly suggest the existence of a profound type of the flavor symmetry including the R-parity. It is plausible that the flavor symmetry also controls the mass spectra of heavy particles which appear on the way between the string scale and the electroweak scale. In order to explore the path for connecting the string scale physics with the low-energy physics, it is necessary for us to study these spectra of heavy particles. For this purpose we examined particle spectra in the context of the $SU(6) \times SU(2)_R$ string-inspired model with the flavor symmetry $Z_{19} \times Z_{18} \times \tilde{D}_4$. In our model the gauge symmetry is spontaneously broken in two steps as

$$SU(6) \times SU(2)_R \xrightarrow{\langle \phi_0 \rangle} SU(4)_P \times SU(2)_L \times SU(2)_R \xrightarrow{\langle \psi_0 \rangle} G_{SM},$$

where $|\langle \phi_0 \rangle| \sim 4.5 \times 10^{17}$ GeV and $|\langle \psi_0 \rangle| \sim 5.2 \times 10^{16}$ GeV. It was shown that there appear characteristic patterns of spectra also in the intermediate energy region. Afterward, we gave the two-loop RG equations of the gauge couplings in the intermediate region, in which each heavy particle decouples by its own stage. The RG runnings of the gauge couplings were studied up to two-loop order and we have obtained consistent gauge couplings at $M_Z$ with the experimental values by taking the reasonable values for the available parameters. We explored solutions in which $SU(3)_c$ and $SU(2)_L$ gauge couplings meet at $\mathcal{O}(5 \times 10^{17}$ GeV) and found a solution of the gauge coupling unification by adjusting the spectra of the anti-generation matter fields. The solution represents the connection between the string scale physics and the electroweak scale physics. It should be emphasized that in our model the unification scale is around the string scale ($\sim 10^{18}$ GeV) but not the so-called GUT scale ($\sim 2 \times 10^{16}$ GeV).

In the present analysis the gauge couplings at the string scale $M_S = 1.5 \times 10^{18}$ GeV are numerically taken as

$$\alpha_6(t = 0) = 0.057, \quad \alpha_2(t = 0) = 0.083. \quad (70)$$

This means that the gauge couplings of $SU(6)$ and $SU(2)_R$ are not unified in the four-dimensional effective theory. In the framework of the higher-dimensional underlying theory, however, the gauge unification of $SU(6)$ and $SU(2)_R$ is expected to be realized non-perturbatively. There is a possibility that the $SU(6)$ and $SU(2)_R$ gauge groups live in distinct world-volumes of different D-branes. It has been pointed out that if the $SU(6)$ gauge group lives in the world-volume of 9-branes and the $SU(2)_R$ in the world-volume of 5-branes, or vice versa, unification solutions can be found in the vicinity of the self-dual point in the moduli space.\(^{24}\)

In the present study we parameterized the magnitude of the effective Yukawa interactions $\bar{\nu}^3$ to adjust the spectra of the anti-generation fields. In order to determine the spectra of the anti-generation fields, we need to know the flavor charges of the Kähler class moduli fields. To this end, it is necessary to elucidate the origin of the flavor symmetry linked to the structure of the compact space. Interestingly, in the case of the four-generation model we were able to obtain the flavor symmetry
and to calculate the flavor charges of the matter superfields, explicitly. In this case
the compact space is a quintic hypersurface in $CP^4$. String compactification on this
hypersurface corresponds to a deformation of the $3^5$ Gepner model. In addition,
it is considered that the flavor symmetry has its origin not only in the symmetric
structure of the compact space but also in the non-commutativity in the compact
space. In the context of M-theory on $G_2$ holonomy spaces, the gauge symmetry
at the string scale is related to the singularity structure of the compact space and
constrained by the cohomology condition on the brane configuration. In this
context the flavor symmetry might be also constrained depending on the singularity
structure of the compact space. The flavor symmetry is expected to provide an
important clue to the study of the path for connecting the string scale physics with
the low-energy physics.

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Appendix

By using the effective superpotential Eq. (13), we can extract the relevant
Yukawa terms which give the dominant contributions to the RG equations.

Due to the symmetry breaking of $SU(6) \times SU(2)_R$ down to $SU(4)_{PS} \times SU(2)_L \times
SU(2)_R$, the matter superfields $\phi(15, 1)$ and $\psi(6^*, 2)$ are decomposed as

$$
\phi(15, 1) \rightarrow \phi(4, 2, 1) + \phi(6, 1, 1) + \phi(1, 1, 1),
$$

$$
\psi(6^*, 2) \rightarrow \psi(4^*, 1, 2) + \phi(1, 2, 2),
$$

(A.1)

respectively in the region between $x^{0.5} M_1$ and $x^{10.125} M_1$. Therefore, the effective
superpotential is expressed as

$$
W_V^{(eff)} = \frac{1}{2} f^{(Z)} S \phi(6,1,1)^2 + \frac{1}{2} f^{(H_0)} S \psi(1,2,2)^2
$$

$$
+ \frac{1}{2} f^{(Z)} S \phi(6,1,1)_i \phi(6,1,1)_j + \frac{1}{2} f^{(H)} S \psi(1,2,2)_i \psi(1,2,2)_j
$$

$$
+ M_{ij}^{(1)} \psi(1,2,2)_0 \phi(4,2,1) \psi(4^*,1,2)_j
$$

$$
+ M_{ij}^{(2)} \psi(4^*,1,2)_0 \phi(4,2,1) \psi(1,2,2)_j
$$

$$
+ M_{ij}^{(3)} \psi(4^*,1,2)_0 \phi(6,1,1) \psi(4^*,1,2)_j
$$

$$
+ M_{ij}^{(4)} \psi(1,2,2)_0 \phi(1,1,1) \psi(1,2,2)_j,
$$

(A.2)

where the effective Yukawa couplings are of the forms

$$
f^{(Z)} = \frac{(2z + 1)}{\sqrt{2}} x \bar{x} \bar{\xi}, \quad f^{(H_0)} = \frac{(2 \eta_0 + 1)}{\sqrt{2}} h_0 x^{\eta_0},
$$
\[ f_{ij}^{(Z)} = \frac{(2\zeta_{ij} + 1)}{\sqrt{2}} z_{ij} x^{\zeta_{ij}}, \quad f_{ij}^{(H)} = \frac{(2\eta_{ij} + 1)}{\sqrt{2}} h_{ij} x^{\eta_{ij}} \]  
\[ (A.3) \]

and
\[ M_{ij}^{(a)} = m_{ij}^{(a)} x^{\mu_{ij}}. \quad (a = 1, 2, 3, 4) \]  
\[ (A.4) \]

As to the last equation, note that \( M_{ij}^{(a)} (a = 1 \sim 4) \) evolves separately, but
\[ M_{ij}^{(1)} = M_{ij}^{(2)} = M_{ij}^{(3)} = M_{ij}^{(4)} = m_{ij} x^{\mu_{ij}} \]  
\[ (A.5) \]

at scale \( |\langle \phi_0 \rangle| = x^{0.5} M_1 \). As seen in Eq. (8), the dominant contributions in the effective potential come from the terms with small powers of \( x \). It turns out that such terms are the effective Yukawa interactions with \( f_{33}^{(Z)}, f_{33}^{(H)} \) and \( M_{33}^{(1 \sim 4)} \) in Eq. (A.2). The RG equations for the Yukawa couplings are given by
\[ \frac{dy_{ij}^{(Z)}}{dt} = \frac{1}{2\pi} \left[ -10\alpha_4 + 8y_{ij}^{(Z)} + 2y_{ij}^{(H)} + 4y_{ij}^{(3)} \right] y_{ij}^{(Z)}, \]  
\[ \frac{dy_{ij}^{(H)}}{dt} = \frac{1}{2\pi} \left[ -3\alpha_2L - 3\alpha_2R + 6y_{ij}^{(Z)} + 4y_{ij}^{(H)} + 8y_{ij}^{(2)} + 2y_{ij}^{(4)} \right] y_{ij}^{(H)}, \]  
\[ \frac{dy_{ij}^{(1)}}{dt} = \frac{1}{2\pi} \left[ -\frac{15}{2} \alpha_4 - 3\alpha_2L - 3\alpha_2R + 8y_{ij}^{(1)} + 2y_{ij}^{(2)} + 3y_{ij}^{(3)} + y_{ij}^{(4)} \right] y_{ij}^{(1)}, \]  
\[ \frac{dy_{ij}^{(2)}}{dt} = \frac{1}{2\pi} \left[ -\frac{15}{2} \alpha_4 - 3\alpha_2L - 3\alpha_2R + y_{ij}^{(H)} + 2y_{ij}^{(1)} + 8y_{ij}^{(2)} + 3y_{ij}^{(3)} + y_{ij}^{(4)} \right] y_{ij}^{(2)}, \]  
\[ \frac{dy_{ij}^{(3)}}{dt} = \frac{1}{2\pi} \left[ -\frac{25}{2} \alpha_4 - 3\alpha_2L + y_{ij}^{(Z)} + 2y_{ij}^{(1)} + 2y_{ij}^{(2)} + 7y_{ij}^{(3)} \right] y_{ij}^{(3)}, \]  
\[ \frac{dy_{ij}^{(4)}}{dt} = \frac{1}{2\pi} \left[ -3\alpha_2L - 3\alpha_2R + y_{ij}^{(H)} + 4y_{ij}^{(1)} + 4y_{ij}^{(2)} + 6y_{ij}^{(4)} \right] y_{ij}^{(4)}. \]  
\[ (A.6) \]

In the region between \( x^{10.125} M_1 \) and \( x^{151} M_1 \), where the gauge group is \( G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y \), the matter superfields are further decomposed as
\[ \phi(4, 2, 1) \rightarrow \phi(3, 2, 1/3) + \phi(1, 2, -1), \]
\[ \phi(6, 1, 1) \rightarrow \phi(3, 1, -2/3) + \phi(3^*, 1, 2/3), \]
\[ \phi(1, 1, 1) \rightarrow \phi(1, 1, 0), \]
\[ \psi(4^*, 1, 2) \rightarrow \psi(3^*, 1, -4/3) + \phi(3^*, 1, 2/3) + \psi(1, 1, 2) + \phi(1, 1, 0), \]
\[ \psi(1, 2, 2) \rightarrow \psi(1, 2, 1) + \phi(1, 2, -1). \]  
\[ (A.7) \]

In this region we obtain five dominant Yukawa terms
\[ M_{33}^{(1U)} \bar{H}_{u0} \tilde{Q}_3 \tilde{U}_3, \quad M_{33}^{(1D)} \bar{H}_{d0} \tilde{Q}_3 \tilde{g}_3, \]
\[ \frac{1}{\sqrt{2}} M_{33}^{(3N)} \tilde{N}^c \tilde{g}_3 \tilde{g}_3, \quad M_{33}^{(3D)} \tilde{D}_6 \tilde{g}_3 \tilde{N}_3 \]

and
\[ f_{33}^{(Z)} \tilde{S} \tilde{g}_3 \tilde{g}_3 = \left( x^{4.375} f_{33}^{(Z)} + \frac{1}{\sqrt{2}} M_{33}^{(3)} \right) \tilde{S} \tilde{g}_3 \tilde{g}_3. \]  
\[ (A.8) \]
We use here the notation "\(\sim\)" for mass eigenstates. The last equation in the above comes from the \(g^\prime-D^c\) mixing. The RG equations for these Yukawa couplings are given by

\[
\begin{align*}
\frac{dy^{(1U)}}{dt} &= \frac{1}{2\pi} \left[ -\frac{16}{3} \alpha_3 - 3\alpha_{2L} - \frac{13}{15} \alpha_1 + 6y^{(1U)} + y^{(1D)} \right] y^{(1U)}, \\
\frac{dy^{(1D)}}{dt} &= \frac{1}{2\pi} \left[ -\frac{16}{3} \alpha_3 - 3\alpha_{2L} - \frac{7}{15} \alpha_1 + y^{(1U)} + 6y^{(1D)} + \frac{1}{2} y^{(3N)} + y^{(Z)} \right] y^{(1D)}, \\
\frac{dy^{(3N)}}{dt} &= \frac{1}{2\pi} \left[ -\frac{16}{3} \alpha_3 - \frac{4}{15} \alpha_1 + 2y^{(1D)} + 5y^{(3N)} + y^{(3D)} + 2y^{(Z)} \right] y^{(3N)}, \\
\frac{dy^{(3D)}}{dt} &= \frac{1}{2\pi} \left[ -\frac{16}{3} \alpha_3 - \frac{4}{15} \alpha_1 + \frac{1}{2} y^{(3N)} + 5y^{(3D)} + y^{(Z)} \right] y^{(3D)}, \\
\frac{dy^{(Z)}}{dt} &= \frac{1}{2\pi} \left[ -\frac{16}{3} \alpha_3 - \frac{4}{15} \alpha_1 + 2y^{(1D)} + y^{(3N)} + y^{(3D)} + 5y^{(Z)} \right] y^{(Z)}. \quad (A.9)
\end{align*}
\]

References

   http://lepewwg.web.cern.ch/LEPEWWG/
25) See, for example, B. S. Acharya, Class. Quantum Grav. 19 (2002), 5619.