Charge Generation in an Oscillating Background

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Preheating after inflation, which can be interpreted as particle creation in an oscillating inflaton background, represents a state far from thermal equilibrium. We extend the field theoretical treatment of the preheating by Linde et al. to the case of multicomponent complex scalars to show that charges are created in this process if $C$ and $CP$ are violated. A new possibility for baryogenesis based on this mechanism is also discussed.

§1. Introduction

In the scenario of inflationary cosmology, which could explain the unsolved problems left by the standard big bang scenario, all the particles at present are thought to be created in the reheating process following the exponentially inflating de Sitter phase. This process is viewed as the decay of the inflaton field, which begins to oscillate around the minimum of the inflaton potential. The inflaton itself is treated as a coherently oscillating classical field, and its oscillation is damped by its decay, with decay products filling the universe to create the hot universe. Several years ago, it was pointed out by Linde et al. that light bosons could be produced in greater numbers than in the perturbative reheating in a different process if some conditions are satisfied with respect to their coupling to the inflaton and the amplitudes and frequency of the inflaton oscillation. This process is called preheating, to distinguish it from the perturbative reheating.

Preheating is nothing but particle creation in the oscillating background. Although particle creation or annihilation generally occurs in a time-dependent background, the number of created particles grows exponentially when the background is periodic in time. This is a kind of parametric resonance. This process of particle creation is far from equilibrium, and the state that results has large quantum fluctuation composed of low frequency modes, whose relaxation process to the equilibrium is also non-equilibrium in nature. Hence, there are chances for generating the matter-antimatter asymmetry of the universe in these eras if $C$ and $CP$ are violated during these processes. In fact, preheating is involved in the generation of the baryon asymmetry of the universe (BAU), focusing on the fact that the low-frequency quantum fluctuations of the created particles are so large that the

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sphaleron processes may be enhanced. In addition to this possibility, it is natural to think that the baryon number and other charges were generated during the preheating, when all the particles in the universe were created. We investigate this possibility for the generation of a charge asymmetry in the presence of $C$ and $CP$ violation in the oscillating background.

We study particle creation when $C$ and $CP$ are violated in the interaction of quantum scalar fields with the background. We extend the analysis by Linde et al.\textsuperscript{3)} to a system of multicomponent complex scalar fields. The interaction with the oscillating background is approximated by successive scattering with some potential, while the effects of $CP$ violation are evaluated perturbatively. In Ref. 3), the number density of the created particles during the preheating is expressed in terms of the wave function in the inflaton background, and the results are reproduced using a successive scattering approximation. It is, however, difficult to distinguish a positive-frequency wave from the negative-frequency wave in the oscillating background, so that one cannot define a charge (to which the contributions from the antiparticles have opposite sign relative to the particles) in terms of wave functions. For this reason the treatment based on successive Bogoliubov transformations is useful. In §2, we generalize the field theory of a real scalar in the oscillating background studied in Ref. 3) to the case of multicomponent complex scalars and formulate the descent equations for the Bogoliubov coefficients. We apply the method to a simple toy model and find some charges to be generated in §3. The final section is devoted to discussion.

§2. Field theoretical treatment

Consider a system of $n$ complex scalar fields $\chi_a$ interacting with an inflaton $\phi$ through the effective Lagrangian

$$\mathcal{L} = \partial_\mu \chi_a^* \partial^\mu \chi_a - m^2_a(t) \chi_a^* \chi_a - \chi_a^* V_{ab}(t) \chi_b - \frac{1}{2} \left( \chi_a W_{ab}(t) \chi_b + \text{c.c.} \right), \tag{2.1}$$

where $m^2_a(t) = g_a^2 \phi^2(t)$ and $a, b = 1, 2, \ldots, n$. The coefficients $V_{ab}(t)$ and $W_{ab}(t)$ represent the collective effects of the oscillating background and interactions with other fields. We treat their effects as perturbations. For simplicity, we consider the case of undamped background oscillations, with the parameters in the broad resonance regime, that is, we assume that the inflaton field is given by

$$\phi(t) = \Phi \sin mt, \tag{2.2}$$

where $m$ is the inflaton mass and $\Phi$ is the constant amplitude, and we neglect the redshift of the frequency of each mode. In this case, each mode of the unperturbed wave functions satisfies the mode equation

$$\left( \partial_t^2 + k^2 + g_a^2 \phi^2 \sin^2 mt \right) \chi_{ak}(t) = 0. \tag{2.3}$$

It is known that wave functions in any periodic potential belong to either stability or instability bands.\textsuperscript{4)} The solutions to (2.3) are given by the Mathieu functions, which
have this property. Among the solutions, those in the instability bands exhibit an exponential increase of particle number. In particular, for \( q_a \equiv \frac{g_a^2 \phi^2}{4m^2} \gg 1 \), some modes are in the broad instability bands, so that they continue to grow for a long time, even if we take the effects of the expanding universe into account. Further, the frequencies of the modes in the broad resonance band are much larger than those of the inflaton oscillation. Then, the particle number changes only when the inflaton field vanishes, while the \( \chi \)-fields evolve adiabatically for time intervals with \( |\phi(t)| \neq 0 \). This observation motivates a study of preheating in which the oscillating background is approximated as a series of successive scatterings by some simple potential. In Ref. 3), near its zero, the inflaton field is replaced by a quadratic potential, and the descent equation for the particle number density in some mode is derived. The particle number density is given in terms of the wave function, which is a solution to (2.3) with any boundary condition. With regard to the charge density, we must distinguish a particle from an antiparticle. This amounts to imposing boundary conditions defining either positive- or negative-frequency modes. Here we express the particle number density and the charge density in terms of the Bogoliubov coefficients, which relate the operators before the inflaton begins to oscillate and those in the adiabatic time intervals.

2.1. Descent equations for the Bogoliubov coefficients

We assume that particles are generated only in the short time interval near \( t_j = \pi j / m_a \), where \( j \) is a non-negative integer, and that the violation of charge conservation and \( CP \) symmetry is effective only within the short interval during which \( m_a(t) \approx 0 \). Hence, for \( t \) with \( t_{j-1} \ll t \ll t_j \), \( \chi_{ak}(t) \) satisfies the unperturbed equation (2.3). Let \( f_{ak}^j(t) \) be the properly normalized positive-frequency solution to (2.3) in the adiabatic interval between \( t_{j-1} \) and \( t_j \). Then the \( \chi \)-field operator is expanded as

\[
\chi_a(x) = \int d^3k \left( a_{ak}^j f_{ak}^j(t) e^{ikx} + b_{ak}^j f_{ak}^{j*}(t) e^{-ikx} \right).
\]  

Without the perturbation, upon scattering by the potential \( g_0^2 \phi^2(t) \) at \( t_j \), the positive-frequency wave \( f_{ak}^j(t) \) is transmitted to a linear combination of \( f_{ak}^{j+1}(t) \) and \( f_{ak}^{j+1*}(t) \), whose coefficients are determined by solving the scattering problem with the potential at \( t_j \). In the presence of the perturbations \( V_{ab} \) and \( W_{ab} \), the scattered wave is composed of various components of \( f_{ak}^{j+1}(t) \) and \( f_{ak}^{j+1*}(t) \). Suppose that the positive-frequency wave \( f_{ak}^0(t) \) for \( t < t_0 \) yields a set of the positive- and negative-frequency waves in the \( j \)-th interval,

\[
\begin{pmatrix}
0 \\
\vdots \\
f_{ak}^0(t) \\
\vdots \\
0
\end{pmatrix} \longrightarrow \cdots \rightarrow \begin{pmatrix}
\alpha_{a1}^j f_{1k}^j(t) + \beta_{a1}^j f_{1k}^{j*}(t) \\
\vdots \\
\alpha_{ab}^j f_{bk}^j(t) + \beta_{ab}^j f_{bk}^{j*}(t) \\
\vdots \\
\alpha_{an}^j f_{nk}^j(t) + \beta_{an}^j f_{nk}^{j*}(t)
\end{pmatrix},
\]  

(2.5)
while the negative-frequency wave $f_{ak}^{0*}(t)$ yields as

\[
\begin{pmatrix}
0 \\
\vdots \\
f_{ak}^{0*}(t) \\
\vdots \\
0
\end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix}
\tilde{\beta}_{a1}^{j} f_{1k}^{j}(t) + \tilde{\alpha}_{a1}^{j} f_{1k}^{j*}(t) \\
\vdots \\
\tilde{\beta}_{ab}^{j} f_{bk}^{j}(t) + \tilde{\alpha}_{ab}^{j} f_{bk}^{j*}(t) \\
\vdots \\
\tilde{\beta}_{an}^{j} f_{nk}^{j}(t) + \tilde{\alpha}_{an}^{j} f_{nk}^{j*}(t)
\end{pmatrix},
\]  

(2.6)

where $\alpha_{ab}^{j} \neq \tilde{\alpha}_{ab}^{j}$ and $\beta_{ab}^{j} \neq \tilde{\beta}_{ab}^{j}$ because of CP violation. Then the Bogoliubov transformation between the creation and annihilation operators for $t < t_0$ and those for $t_{j-1} < t < t_j$ is

\[
a_{ak}^{j} = a_{bk}^{0} \alpha_{ba}^{j} + b_{bk}^{0*} \tilde{\beta}_{ba}^{j}, \\
b_{ak}^{j*} = a_{bk}^{0*} \beta_{ba}^{j} + b_{bk}^{0} \tilde{\alpha}_{ba}^{j}.
\]  

(2.7)

Requiring

\[
[a_{ak}^{j}, a_{bk}^{k*}] = [b_{ak}^{j}, b_{bk}^{k*}] = \delta_{ab} \delta^3(k - k'), \\
[a_{ak}^{j}, b_{bk}^{k*}] = 0,
\]  

(2.8)

we find the following conditions on the Bogoliubov coefficients:

\[
\alpha_{ca}^{j} \alpha_{cb}^{j*} - \beta_{ca}^{j} \tilde{\beta}_{cb}^{j*} = \delta_{ab}, \\
\tilde{\alpha}_{ca}^{j*} \alpha_{cb}^{j} - \beta_{ca}^{j*} \beta_{cb}^{j} = \delta_{ab}, \\
\tilde{\alpha}_{ca}^{j} \beta_{cb}^{j*} - \tilde{\beta}_{ca}^{j} \alpha_{cb}^{j*} = 0.
\]  

(2.9)

In terms of the $n \times n$ matrix notation, $\alpha^j = (\alpha_{ab}^j)$, $\beta^j = (\beta_{ab}^j)$, etc., these conditions are expressed as

\[
\alpha^{j*} \alpha^j - \tilde{\beta}^{j*} \tilde{\beta}^j = \tilde{\alpha}^{j*} \tilde{\alpha}^j - \beta^{j*} \beta^j = 1, \\
\beta^{j*} \alpha^j - \tilde{\alpha}^{j*} \tilde{\alpha}^j = 0.
\]  

(2.10)

We can also show that

\[
\alpha^{j*} \beta^j - \tilde{\beta}^{j*} \tilde{\beta}^j = \alpha^{j*} \alpha^j - \tilde{\beta}^{j*} \tilde{\beta}^j = 1, \\
\alpha^j \tilde{\beta}^{j*} - \tilde{\alpha}^j \tilde{\alpha}^{j*} = 0.
\]  

(2.11)

Now we define the number density of the particles with momentum $k$ in the $j$-th interval as

\[
n^j_k \equiv \frac{1}{V} \langle 0^0 | \sum_{a=1}^n (a_{ak}^{j*} a_{ak}^{j} + b_{ak}^{j*} b_{ak}^{j}) | 0^0 \rangle,
\]  

(2.12)

where $| 0^0 \rangle$ is defined by $a_{ak}^{0*} | 0^0 \rangle = b_{ak}^{0} | 0^0 \rangle = 0$ and $V = \delta^3(k)$. By use of (2.7), we have

\[
n^j_k = \sum_{a,b} (\tilde{\beta}_{ba}^{j*} \tilde{\beta}_{ba}^{j} + \beta_{ba}^{j*} \beta_{ba}^{j}) = \text{Tr} \left( \tilde{\beta}^{j*} \tilde{\beta}^j + \beta^{j*} \beta^j \right).
\]  

(2.13)

If each component of $\phi_a(x)$ has $U(1)$ charge $Q_a$, the density of the generated charge for mode $k$ in the $j$-th interval is given by

\[
x^j_k \equiv \frac{1}{V} \langle 0^0 | \sum_{a=1}^n Q_a (a_{ak}^{j*} a_{ak}^{j} - b_{ak}^{j*} b_{ak}^{j}) | 0^0 \rangle,
\]  

(2.14)
which can be written in terms of the Bogoliubov coefficients as

$$\hat{j}_k^j = \sum_{a,b} Q_a \left( \beta_{ba}^{j*} \beta_{ba}^j - \beta_{ba}^j \beta_{ba}^{j*} \right) = \text{Tr} \left[ Q \left( \beta_{ba}^{j*} \beta_{ba}^j - \beta_{ba}^j \beta_{ba}^{j*} \right) \right],$$

(2.15)

where $Q$ on the right-hand side is the $n \times n$ matrix defined as $Q = \text{diag}(Q_1, Q_2, \cdots, Q_n)$.

In the case $n = 1$, it is easy to show $|\beta_j|^2 = |\beta_{ba}^j|^2$ by use of (2.10), so that $j_k^j = 0$. Hence, we need at least two complex scalars interacting with the oscillating background in order to generate charge asymmetry. This fact can be understood as follows: If we regard the particles and the charges as the decay products of the oscillating field, at least two decay channels are needed to produce a nonzero charge, since the effect of $CP$ violation appears as the interference of the two channels. This is the same situation as in the GUT baryogenesis through heavy boson decay.

The relation between the Bogoliubov coefficients in the $(j+1)$-th interval and those in the $j$-th interval can be found by solving the scattering problem with the potential at $t_j$, just as done in Ref. 3). Within each adiabatic interval, the positive frequency wave function $f_{ak}^j(t)$ is approximated well by

$$h_{ak}(t) = \frac{1}{\sqrt{2\omega_a(t)}} e^{-i \int_0^t dt' \omega_a(t')},$$

(2.16)

where

$$\omega_a(t) = \sqrt{k^2 + m_a^2(t)} = \sqrt{k^2 + g_a^2 \phi^2 \sin^2 mt},$$

(2.17)

up to some constant phase. For $t$ near $t_j$, where the adiabatic approximation is no longer valid, we solve the scattering problem by employing some approximate form of the potential at $t_j$. We adopt another type of potential which allows us to define an asymptotic state, given by $\omega_a(t) \simeq \omega_a^0 = \text{const}$ for $|t - t_j| \gg 1/m$, in contrast to the quadratic potential $m_a^2(t) \simeq m^2(t - t_j)^2$ used in Ref. 3). This choice of the potential is more appropriate for the perturbative calculation of the $CP$-violating scattering presented in the next subsection. The general formulas presented in this subsection are independent of the detailed form of the potential. If we use this potential, the adiabatic wave (2.16) behaves, far from the potential, as

$$h_{ak}(t) \rightarrow \frac{1}{\sqrt{2\omega_a^0}} e^{-i \omega_a^0(t-t_j) - i \theta_{ak}^+-i \theta_{ak}^-}, \quad (\text{as } t-t_j \rightarrow \pm \infty)$$

(2.18)

where $\theta_{ak}^j = \int_{t_0}^{t_j} dt' \omega_a(t')$ is the phase accumulated by the moment $t_j$, and $\theta_{ak}^+(\theta_{ak}^-)$ is the phase difference between $\int_{t_0}^{t_j} dt' \omega_a(t')$ and $\omega_a^0(t-t_j)$ for $t \gg t_j$ ($t \ll t_j$). Then, the positive-frequency wave for $t < t_0$, $f_{ak}^{t_0}(t)$, can be approximated by $h_{ak}(t)$, while $f_{ak}^j(t)$ in the $j$-th adiabatic interval can be expressed as a linear combination of $h_{bk}(t)$ and $h_{bk}^*(t)$.

Suppose that we find solutions to the wave equations (usually, using a perturbative method) satisfying the boundary condition that only the positive-frequency
wave in the \(a\)-th component exists for \(t \ll t_j\),

\[
\begin{pmatrix}
0 \\
\vdots \\
0
\end{pmatrix} \leftrightarrow \begin{pmatrix}
\chi_{1k}(t) \\
\vdots \\
\chi_{nk}(t)
\end{pmatrix} \quad \begin{pmatrix}
\mathcal{A}_a \frac{1}{\sqrt{2\omega_0^a}} e^{-i\omega_0^a(t-t_j)} + B_a \frac{1}{\sqrt{2\omega_0^a}} e^{i\omega_0^a(t-t_j)} \\
\vdots \\
\mathcal{A}_n \frac{1}{\sqrt{2\omega_0^n}} e^{-i\omega_0^n(t-t_j)} + B_n \frac{1}{\sqrt{2\omega_0^n}} e^{i\omega_0^n(t-t_j)}
\end{pmatrix},
\]

and similarly solutions satisfying the boundary condition that only the negative-frequency wave exists for \(t \ll t_j\),

\[
\begin{pmatrix}
0 \\
\vdots \\
0
\end{pmatrix} \leftrightarrow \begin{pmatrix}
\chi_{1k}(t) \\
\vdots \\
\chi_{nk}(t)
\end{pmatrix} \quad \begin{pmatrix}
\check{\mathcal{B}}_a \frac{1}{\sqrt{2\omega_0^a}} e^{-i\omega_0^a(t-t_j)} + \check{\mathcal{A}}_a \frac{1}{\sqrt{2\omega_0^a}} e^{i\omega_0^a(t-t_j)} \\
\vdots \\
\check{\mathcal{B}}_n \frac{1}{\sqrt{2\omega_0^n}} e^{-i\omega_0^n(t-t_j)} + \check{\mathcal{A}}_n \frac{1}{\sqrt{2\omega_0^n}} e^{i\omega_0^n(t-t_j)}
\end{pmatrix},
\]

Here the coefficients \(\mathcal{A}, \check{\mathcal{B}}, \check{\mathcal{A}}\) and \(\check{\mathcal{B}}\), which are also kinds of Bogoliubov coefficients, satisfy the same relations as \(\alpha, \beta, \check{\alpha}\) and \(\check{\beta}\):

\[
\mathcal{A}^\dagger \mathcal{A} - \check{\mathcal{B}}^\dagger \check{\mathcal{B}} = \check{\mathcal{A}}^\dagger \check{\mathcal{A}} - \mathcal{B}^\dagger \mathcal{B} = 1, \quad \mathcal{B}^\dagger \mathcal{A} - \check{\mathcal{A}}^\dagger \check{\mathcal{B}} = 0, \\
\mathcal{A} \check{\mathcal{A}}^\dagger - \check{\mathcal{B}} \mathcal{B}^\dagger = \check{\mathcal{A}} \check{\mathcal{A}}^\dagger - \mathcal{B} \check{\mathcal{B}}^\dagger = 1, \quad \mathcal{B} \check{\mathcal{A}}^\dagger - \check{\mathcal{B}} \mathcal{B}^\dagger = 0.
\]

Inverting (2.5) and (2.6) and replacing \(f_{ak}^0(t)\) with \(h_{ak}(t)\), we have

\[
\begin{pmatrix}
0 \\
\vdots \\
0
\end{pmatrix} \sim \begin{pmatrix}
(\alpha^{ij}) \, h_{1k}(t) - (\beta^{ij}) \, h_{1k}^*(t) \\
\vdots \\
(\alpha^{ij}) \, h_{nk}(t) - (\beta^{ij}) \, h_{nk}^*(t)
\end{pmatrix},
\]

and

\[
\begin{pmatrix}
0 \\
\vdots \\
0
\end{pmatrix} \sim \begin{pmatrix}
(\check{\alpha}^{ij}) \, h_{1k}^*(t) - (\check{\beta}^{ij}) \, h_{1k}(t) \\
\vdots \\
(\check{\alpha}^{ij}) \, h_{nk}^*(t) - (\check{\beta}^{ij}) \, h_{nk}(t)
\end{pmatrix}.
\]

Since the manners in which \(h_{ak}(t)\) and \(h_{ak}^*(t)\) appearing in each element on the right-hand sides of (2.22) and (2.23) are scattered by the potential at \(t_j\) are determined by
(2.19) and (2.20), respectively, the \( b \)-th component of the right-hand side of (2.22) becomes, after the scattering \((t \gg t_j)\),

\[
\left[ (\alpha^{j+1})_a \right]_{ac} e^{-i(\theta^j_{ck} + \theta^j_{ck})} A_{cb} - \left[ (\tilde{\beta}^{j+1})_a \right]_{ac} e^{i(\theta^j_{ck} + \theta^j_{ck})} B_{cb} \right) e^{i(\theta^j_{bk} + \theta^j_{bk})} h_{bk}(t)
- \left[ (\beta^{j+1})_a \right]_{ac} e^{-i(\theta^j_{ck} + \theta^j_{ck})} \tilde{A}_{cb} - \left[ (\alpha^{j+1})_a \right]_{ac} e^{-i(\theta^j_{ck} + \theta^j_{ck})} B_{cb} \right) e^{-i(\theta^j_{bk} + \theta^j_{bk})} h^*_b(t). \tag{2.24}
\]

This should be identified with \( (\alpha^{j+1})_a \) \( h_{bk}(t) \) \(- (\beta^{j+1})_a \) \( h_{bk}^*(t) \), and the \( b \)-th component of the right-hand side of (2.23) becomes

\[
\left[ (\tilde{\alpha}^{j+1})_a \right]_{ac} e^{i(\theta^j_{ck} + \theta^j_{ck})} \tilde{B}_{cb} - \left[ (\beta^j)_a \right]_{ac} e^{-i(\theta^j_{ck} + \theta^j_{ck})} A_{cb} \right) e^{i(\theta^j_{bk} + \theta^j_{bk})} h_{bk}(t)
- \left[ (\beta^j)_a \right]_{ac} e^{-i(\theta^j_{ck} + \theta^j_{ck})} \tilde{B}_{cb} - \left[ (\tilde{\alpha}^{j+1})_a \right]_{ac} e^{i(\theta^j_{ck} + \theta^j_{ck})} \tilde{A}_{cb} \right) e^{-i(\theta^j_{bk} + \theta^j_{bk})} h_{bk}^*(t). \tag{2.25}
\]

This should be identified with \( (\tilde{\alpha}^{j+1})_a \) \( h_{bk}^*(t) \) \(- (\beta^j)_a \) \( h_{bk}(t) \). If we denote the \( n \times n \) matrix of the phase factors as

\[
U_{\pm} = \text{diag} \left( e^{-i(\theta^j_{ik} + \theta^j_{ik})}, \ldots, e^{-i(\theta^j_{nk} + \theta^j_{nk})} \right), \tag{2.26}
\]

we obtain the descent equations for the Bogoliubov coefficients:

\[
\alpha^{j+1} = U_+ \left( A \right) \alpha^j - B \tilde{B} \alpha^j, \\
\beta^{j+1} = U_+ \left( A \right) \beta^j - B \beta^j, \\
\tilde{\alpha}^{j+1} = U_+ \left( \tilde{A} \right) \tilde{\alpha}^j - B \tilde{B} \beta^j, \\
\tilde{\beta}^{j+1} = U_+ \left( \tilde{A} \right) \beta^j - B \tilde{B} \tilde{\alpha}^j. \tag{2.27}
\]

As a check, it can be shown that these \((j + 1)\)-th Bogoliubov coefficients satisfy the same relations as (2.10) and (2.11).

2.2. **Perturbative calculation of the scattering matrices**

Now we calculate the scattering matrices \( A, \tilde{B} \) employing the perturbative method. The wave equations to be solved are written as

\[
\left[ \left( \partial_t^2 + k^2 + m^2_a(t) \right) 1 + \left[ \begin{array}{cc} V_{ab}(t) & W_{ab}^*(t) \\ W_{ab}(t) & V_{ab}^*(t) \end{array} \right] \right] \left( \begin{array}{c} \chi_{bk}(t) \\ \chi_{bk}^*(t) \end{array} \right) = 0. \tag{2.28}
\]

For convenience, let us introduce the following symbols:

\[
X_k(t) = \left( \begin{array}{c} \chi_{ak}(t) \\ \chi_{ak}^*(t) \end{array} \right), \\
\Delta_0 = \left( \partial_t^2 + k^2 + m^2_a(t) \right) 1 = \left( \partial_t^2 + \omega^2_a(t) \right) 1, \\
\Delta_1 = \left( \begin{array}{cc} V_{ab}(t) & W_{ab}^*(t) \\ W_{ab}(t) & V_{ab}^*(t) \end{array} \right). \tag{2.29}
\]

Then a perturbative solution to the equation

\[
(\Delta_0 + \Delta_1) X_k(t) = 0, \tag{2.30}
\]
to first order in $\Delta_1$ is given by
\[ X_k(t) = X_k^{(0)}(t) - \Delta_0^{-1} \Delta_1 X_k^{(0)}(t), \]  
where $X_k^{(0)}(t)$ is a solution to
\[ \Delta_0 X_k^{(0)}(t) = 0. \]

In order to evaluate the scattering matrices $A$ and $B$, we must prepare $X_k(t)$ such that the only positive-frequency wave is $\chi_{ak}(t)$, and the other components vanish in the remote past. We solve the scattering problem with this initial condition for the approximate potential that admits the asymptotic states. Let $f_{ak}(t)$ be a wave function satisfying
\[ \left( \partial_t^2 + \omega_a^2(t) \right) f_{ak}(t) = 0, \quad \text{and} \quad f_{ak}(t) \xrightarrow{t \to -\infty} \frac{1}{\sqrt{2\omega_a^0}} e^{-i\omega_a^0 t}. \]

Suppose that we solve this unperturbed equation and find the expression as
\[ f_{ak}(t) = \alpha_a f_{ak}^{\text{out}}(t) + \beta_a f_{ak}^{\text{out}*}(t), \]
where $f_{ak}^{\text{out}}(t)$ satisfies
\[ \left( \partial_t^2 + \omega_a^2(t) \right) f_{ak}^{\text{out}}(t) = 0, \quad \text{and} \quad f_{ak}^{\text{out}}(t) \xrightarrow{t \to -\infty} \frac{1}{\sqrt{2\omega_a^0}} e^{-i\omega_a^0 t}. \]

According to the general theory of the distorted-wave Born approximation,\(^5\) the Green function $G_a(t, t')$ satisfying
\[ \left( \partial_t^2 + \omega_a^2(t) \right) G_a(t, t') = \delta(t - t'), \]
with a boundary condition such that there is only a positive-frequency wave in the remote past, is given by
\[ G_a(t, t') = \begin{cases} \frac{1}{w} f_{ak}(t)f_{ak}^{*}(t'), & (t < t') \\ \frac{1}{w} f_{ak}(t')f_{ak}(t), & (t' < t) \end{cases} \]
where $w$ is the Wronskian defined by
\[ w \equiv f_{ak}(t)\partial_t f_{ak}^*(t) - f_{ak}^*(t)\partial_t f_{ak}(t), \]
which is time-independent. In our case, $w = i$. Then our $\Delta_0^{-1}$ is
\[ \Delta_0^{-1} = \text{diag} \left( G_1, \cdots, G_n, G_1^*, \cdots, G_n^* \right). \]

As the unperturbed wave $X_k^{(0)}(t)$, we take
\[ X_k^{[a](0)}(t) = \begin{pmatrix} c f_k(t) \\ c^* f_k(t) \end{pmatrix} = \begin{pmatrix} c_{a1} f_{1k}(t) \\ \vdots \\ c_{a1} f_{1k}^*(t) \end{pmatrix}, \]
and choose the set of parameters $c_{ab}$ such that

$$X_k^{[a]}(t) = X_k^{[a]|0\rangle}(t) - \Delta_0^{-1}\Delta_1 X_k^{[a]|0\rangle}(t) \xrightarrow{t \to -\infty} \begin{pmatrix} 0 \\ \vdots \\ f_{ak}(t) \\ \vdots \\ f_{ak}^\ast(t) \\ 0 \end{pmatrix}. \quad (2.41)$$

In terms of the $n$-component wave function, the perturbed wave becomes

$$\chi_k^{[a]}(t) = cf_k(t) - GV(c f_k)(t) - GW^\ast(c^\ast f_k^\ast)(t). \quad (2.42)$$

Imposing the initial condition (2.41), we find that $c_{ab}$ should be taken to satisfy

$$c_{ab} + i c_{ac} \int_{-\infty}^{\infty} dt' V_{bc}(t') f_{bk}^\ast(t') f_{ck}(t') + i c_{ac} \int_{-\infty}^{\infty} dt' W_{bc}^\ast(t') f_{bk}^\ast(t') f_{ck}(t') = \delta_{ab}. \quad (2.43)$$

Once $c_{ab}$ is determined, we obtain the scattering matrices $A$ and $B$ by extracting the coefficients of the positive- and negative-frequency waves, respectively, from (2.42) in the remote future:

$$A_{ab} = c_{ab} \alpha_b + i \beta_b \int_{-\infty}^{\infty} dt' f_{bk}^\ast(t') [V_{bc}(t') c_{ac} f_{ck}(t') + W_{bc}^\ast(t') c_{ac} f_{ck}^\ast(t')] ,$$

$$B_{ab} = c_{ab} \beta_b + i \alpha_b \int_{-\infty}^{\infty} dt' f_{bk}(t') [V_{bc}(t') c_{ac} f_{ck}(t') + W_{bc}^\ast(t') c_{ac} f_{ck}^\ast(t')] . \quad (2.44)$$

Similarly, by preparing the negative-frequency wave in the remote past, we can find the other set of the scattering matrices, $\tilde{A}$ and $\tilde{B}$. They are given by

$$\tilde{A}_{ab} = \tilde{c}_{ab} \alpha_b^\ast - i \beta_b \int_{-\infty}^{\infty} dt' f_{bk}^\ast(t') [V_{bc}(t') \tilde{c}_{ac} f_{ck}^\ast(t') + W_{bc}^\ast(t') \tilde{c}_{ac} f_{ck}(t')] ,$$

$$\tilde{B}_{ab} = \tilde{c}_{ab} \beta_b^\ast - i \alpha_b \int_{-\infty}^{\infty} dt' f_{bk}(t') [V_{bc}(t') \tilde{c}_{ac} f_{ck}(t') + W_{bc}(t') \tilde{c}_{ac} f_{ck}^\ast(t')] , \quad (2.45)$$

where $\tilde{c}_{ab}$ must satisfy

$$\tilde{c}_{ab} - i \tilde{c}_{ac} \int_{-\infty}^{\infty} dt' V_{bc}(t') f_{bk}(t') f_{ck}(t') - i \tilde{c}_{ac} \int_{-\infty}^{\infty} dt' W_{bc}^\ast(t') f_{bk}(t') f_{ck}(t') = \delta_{ab}. \quad (2.46)$$

In the case $W_{ab}(t) = 0$, for which the global $U(1)$ symmetry exists corresponding to the rotation of all $\chi_a(x)$ with a common phase, the scattering matrices are simply given by

$$A = N^{-1} \left( \alpha + K \beta^\dagger \right) , \quad B = N^{-1} \left( \beta + K \alpha^\dagger \right) ,$$

$$\tilde{A} = \tilde{N}^{-1} \left( \alpha^\dagger + \tilde{K} \beta \right) , \quad \tilde{B} = \tilde{N}^{-1} \left( \beta^\dagger + \tilde{K} \alpha \right) . \quad (2.47)$$
where $\alpha$ and $\beta$ are diagonal matrices defined by

$$\alpha = \text{diag}(\alpha_1, \alpha_2, \cdots, \alpha_n), \quad \beta = \text{diag}(\beta_1, \beta_2, \cdots, \beta_n),$$

and the matrices $N$, $\tilde{N}$, $K$ and $\tilde{K}$ are given by

$$N_{ab} = \delta_{ab} + i \int_{-\infty}^{\infty} dt \, V_{ba}(t)f_{ak}(t)f_{bk}^*(t),$$

$$\tilde{N}_{ab} = \delta_{ab} - i \int_{-\infty}^{\infty} dt \, V_{ba}(t)f_{ak}^*(t)f_{bk}(t),$$

$$K_{ab} = i \int_{-\infty}^{\infty} dt \, V_{ba}(t)f_{ak}(t)f_{bk}(t),$$

$$\tilde{K}_{ab} = -i \int_{-\infty}^{\infty} dt \, V_{ba}(t)f_{ak}^*(t)f_{bk}^*(t).$$

Because of the hermiticity of the matrix $V_{ab}(t)$, implying $\tilde{K}^\dagger = K$ and $T_{ab} \equiv \int_{-\infty}^{\infty} dt \, V_{ba}(t)f_{ak}(t)f_{bk}^*(t) = T_{ba}^*$, the matrix $N$ can be regarded as the unitary matrix $\exp(iT)$.

### §3. Example

Once the model of the inflaton interacting with at least two complex scalars is specified, one can evaluate the ‘effective potentials’ $V_{ab}(t)$ and $W_{ab}(t)$, which may be induced directly by coupling with the inflaton to the complex scalars, or through some loop effects including the scalars and other fields. Here, without proposing any specific model, we illustrate calculation of the generated charges and particle numbers, assuming some simple form of the $CP$-violating potentials. For simplicity, we concentrate on the case $W_{ab}(t) = 0$ and $n = 2$. We also assume the degeneracy of the scalar masses $(m_1 = m_2)$ and the property $V_{11} = V_{22}$ of the potential, which may result from some discrete symmetry. Under these assumptions, we have

$$f_{1k}(t) = f_{2k}(t) = f_k(t), \quad \alpha_1 = \alpha_2 = \alpha, \quad \beta_1 = \beta_2 = \beta.$$

Then the matrices in (2.49) and (2.50) are reduced to

$$K = \begin{pmatrix} iK_1 \
 iK_2(1 + i\epsilon_K) \
 \end{pmatrix}, \quad \tilde{K} = K^\dagger,$$

$$N = \begin{pmatrix} 1 + iN_1 \
 iN_2(1 + i\epsilon_N) \
 \end{pmatrix}, \quad \tilde{N} = N^\dagger,$$

where

$$N_1 = \int_{-\infty}^{\infty} dt \, V_{11}(t) |f_k(t)|^2, \quad N_2 = \int_{-\infty}^{\infty} dt \, \text{Re}V_{12}(t) |f_k(t)|^2,$$

$$\epsilon_N = \frac{1}{N_2} \int_{-\infty}^{\infty} dt \, \text{Im}V_{12}(t) |f_k(t)|^2,$$

$$K_1 = \int_{-\infty}^{\infty} dt \, V_{11}(t)f_k^2(t), \quad K_2 = \int_{-\infty}^{\infty} dt \, \text{Re}V_{12}(t)f_k^2(t),$$

$$\epsilon_K = \frac{1}{K_2} \int_{-\infty}^{\infty} dt \, \text{Im}V_{12}(t)f_k^2(t).$$

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Once the explicit forms of $f_k(t)$, $V_{11}(t)$ and $V_{12}(t)$ are obtained, we can calculate all the quantities needed to follow the descent equations (2.27), so that the time evolution of the densities of particle number and charges.

First of all, we prepare the unperturbed wave that is appropriate for the perturbative calculation of the scattering matrices. For this purpose, we substitute the oscillating potential near its zeros with the asymptotically flat potential as

$$\phi^2 \sin^2 mt \simeq 2\phi^2 \tanh^2 \left( \frac{m(t - t_j)}{\sqrt{2}} \right).$$  \hspace{1cm} (3.5)

This is better than the simple quadratic potential adopted in Ref. 3, in that the potential of (3.5) matches to the original oscillating potential up to fourth order in $m(t - t_j)$. Introducing the dimensionless parameters

$$\tau = k_*(t - t_j), \quad \kappa = \frac{k}{k_*}, \quad \xi = \sqrt{\frac{m}{2g\Phi}} = \frac{1}{2q^{1/4}},$$  \hspace{1cm} (3.6)

where $k_* = \sqrt{g\Phi m} = \sqrt{2mq^{1/4}}$ and $q = \frac{g^2 \phi^2}{4m^2}$, the wave equation is expressed as

$$\frac{d^2 f_k(\tau)}{d\tau^2} + \left[ \kappa^2 + \frac{1}{\xi^2} \tanh^2 (\xi \tau) \right] f_k(\tau) = 0.$$  \hspace{1cm} (3.7)

We find that the solution which asymptotically approaches the positive-frequency wave in the remote past is given, in terms of $x \equiv \xi \tau$, by

$$f_k(t) = \frac{1}{\sqrt{2\omega_0}} \left( e^x + e^{-x} \right)^{-ie} F \left( a, b, c; \frac{1}{1 + e^{-2x}} \right) \quad \tau \to -\xi \infty \frac{1}{\sqrt{2\omega_0}} e^{-i\omega_0(t - t_j)},$$  \hspace{1cm} (3.8)

where

$$\epsilon = \frac{g\phi}{m} \sqrt{1 + \frac{k^2}{2g^2\phi^2}} = 2q^{1/2} \sqrt{1 + \frac{\kappa^2}{4q^{1/2}}},$$

$$\omega_0 = 2\epsilon k_* = \sqrt{2m\epsilon},$$

$$a = -4iq^{1/2} \left( \sqrt{1 + \frac{\kappa^2}{4q^{1/2}}} - \sqrt{1 - \frac{1}{64q}} \right) + \frac{1}{2},$$

$$b = -4iq^{1/2} \left( \sqrt{1 + \frac{\kappa^2}{4q^{1/2}}} + \sqrt{1 - \frac{1}{64q}} \right) + \frac{1}{2},$$

$$c = -4iq^{1/2} \sqrt{1 + \frac{\kappa^2}{4q^{1/2}}} + 1,$$  \hspace{1cm} (3.9)

and $F(a, b, c; z)$ is the hypergeometric function. By use of its property, we find, in the limit $t - t_j \to \infty$,

$$f_k(t) \to \frac{\alpha_k}{\sqrt{2\omega_0}} e^{-i\omega_0(t - t_j)} + \frac{\beta_k}{\sqrt{2\omega_0}} e^{i\omega_0(t - t_j)},$$  \hspace{1cm} (3.11)
where
\[
\alpha_k = \frac{\Gamma(-2i\epsilon + 1)\Gamma(-2i\epsilon)}{\Gamma\left(\frac{1}{2} - 2i\epsilon + 4iq^{1/2}\sqrt{1 - \frac{1}{64q}}\right)\Gamma\left(\frac{1}{2} - 2i\epsilon - 4iq^{1/2}\sqrt{1 - \frac{1}{64q}}\right)}.
\]
\[
\beta_k = \frac{\Gamma(-2i\epsilon + 1)\Gamma(2i\epsilon)}{\Gamma\left(\frac{1}{2} + 4iq^{1/2}\sqrt{1 - \frac{1}{64q}}\right)\Gamma\left(\frac{1}{2} - 4iq^{1/2}\sqrt{1 - \frac{1}{64q}}\right)} = -i \frac{\cosh(4\pi q^{1/2}\sqrt{1 - \frac{1}{64q}})}{\sinh(2\pi \epsilon)}.
\] (3.12)

Among the phases defined in (2.18), \(\theta^j_k\) is given by
\[
\theta^j_k = \int_0^{t_j} dt \omega(t) = j \theta_k,
\] (3.13)
with
\[
\theta_k = 2\sqrt{2} q^{1/4} \int_0^{\pi/2} d\varphi \sqrt{\kappa^2 + 2q^{1/2}\sin^2 \varphi},
\] (3.14)
while \(\vartheta_k\) is evaluated by use of the potential (3.5) as
\[
\vartheta_k = 2\epsilon \log \frac{\epsilon}{q^{1/4}\kappa} - 4q^{1/2} \log \frac{q^{-1/4}(\epsilon + 2q^{1/2})}{\kappa}.
\] (3.15)

With these quantities, one can evaluate the particle number density in the absence of the perturbation. The result is the same as that in Ref. 3) for the case of \(q^{1/2} = 1/(4\xi^2) \gg 1\).

For the effective potential, we adopt
\[
V_{11}(t) = V_{22}(t) = 2\lambda_1 \Phi^2 (\tanh^2 \xi \tau - 1),
\]
\[
V_{12}(t) = 2\lambda_2 \Phi^2 e^{i\theta(\tau)} (\tanh^2 \xi \tau - 1),
\] (3.16)
which have the same \(t\) dependence as the potential in (3.7) and vanish for large \(|\tau|\). We specify the \(CP\)-violating phase \(\theta(\tau)\) below, where we discuss the numerical calculation. If we write
\[
l_1 = \frac{\lambda_1}{g^2}, \quad l_2 = \frac{\lambda_2}{g^2},
\] (3.17)
the quantities composing the elements of the matrices (3.2) are given by
\[
N_1 = -\frac{2l_1 q^{1/2}}{\sqrt{1 + \kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \frac{1}{\cosh^2 x} \left| F\left(a, b, c; \frac{1}{1 + e^{-2x}}\right) \right|^2,
\]
\[
N_2 = -\frac{2l_2 q^{1/2}}{\sqrt{1 + \kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \frac{\cos \theta(x)}{\cosh^2 x} \left| F\left(a, b, c; \frac{1}{1 + e^{-2x}}\right) \right|^2,
\]
\[
\epsilon_N = -\frac{1}{N_2} \frac{2l_2 q^{1/2}}{\sqrt{1 + \kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \frac{\sin \theta(x)}{\cosh^2 x} \left| F\left(a, b, c; \frac{1}{1 + e^{-2x}}\right) \right|^2,
\]
\[
K_1 = -\frac{2l_1 q^{1/2}}{\sqrt{1 + k^2 / (4q^{1/2})}} \int_{-\infty}^{\infty} dx \frac{1}{\cosh^2 x} \left( e^x + e^{-x} \right) 4i e^{4i \epsilon} F \left( a, b, c; \frac{1}{1 + e^{-2x}} \right)^2,
\]
\[
K_2 = -\frac{2l_2 q^{1/2}}{\sqrt{1 + k^2 / (4q^{1/2})}} \int_{-\infty}^{\infty} dx \frac{\cos \theta(x)}{\cosh^2 x} \left( e^x + e^{-x} \right) 4i e^{4i \epsilon} F \left( a, b, c; \frac{1}{1 + e^{-2x}} \right)^2,
\]
\[
\epsilon_K = -\frac{1}{K_2} \frac{2l_2 q^{1/2}}{\sqrt{1 + k^2 / (4q^{1/2})}} \int_{-\infty}^{\infty} dx \frac{\sin \theta(x)}{\cosh^2 x} \left( e^x + e^{-x} \right) 4i e^{4i \epsilon} F \left( a, b, c; \frac{1}{1 + e^{-2x}} \right)^2.
\]
(3-18)

Now, we are ready to evaluate the particle number density and charge density. The remaining quantities we must specify are the values of \( q, l_1 \) and \( l_2 \), and the functional form of \( \theta(\tau) \). All the quantities with dimensions of mass are expressed with appropriate powers of the inflaton mass \( m \). For definiteness, we adopt
\[
\theta(x) = \frac{\theta_0}{1 + e^{2x}},
\]
(3-19)
which changes from \( \theta_0 \) to 0 as \( x \) goes from \(-\infty\) to \( \infty \). We set \( q = 200 \), for which the parametric resonance predicted by the Mathieu function exists for a wide range of the momentum \( \kappa \), that is, in broad resonance. We numerically evaluated the scattering matrices (2.47) for a fixed \( \kappa \), and studied the time evolution of
\[
n^j_k = \sum_{a,b=1}^2 \left( |\beta^j_{ab}|^2 + |\tilde{\beta}^j_{ab}|^2 \right),
\]
\[
J^j_{1k} = |\tilde{\beta}^j_{11}|^2 + |\tilde{\beta}^j_{21}|^2 - |\beta^j_{11}|^2 - |\beta^j_{21}|^2,
\]
\[
J^j_{2k} = |\tilde{\beta}^j_{12}|^2 + |\tilde{\beta}^j_{22}|^2 - |\beta^j_{12}|^2 - |\beta^j_{22}|^2,
\]
(3-20)
where (3-20) and (3-21) are the charge densities defined with \( Q_1 = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) \) and \( Q_2 = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) \) in (2.15), respectively, by numerically solving the descent equations (2.27) with the initial conditions \( \alpha^0 = \tilde{\alpha}^0 = 1 \) and \( \beta^0 = \tilde{\beta}^0 = 0 \). Because of the \( U(1) \) symmetry mentioned above (2.47), \( J^j_{1k} + J^j_{2k} = 0 \) in any \( j \)-th interval. Although the particle number grows exponentially for \( \kappa \) in one of the resonance band, it ceases to grow due to the back reaction after a finite number of inflaton oscillations, as studied in Ref. 3). The number of times that the inflaton oscillates before the particle creation ends depends on \( q \) and the coupling constant \( g \). In the present case, one might think that the \( CP \) violating effects affect the back reaction and rescattering. For the back reaction, the \( CP \) violation induces corrections to the \( CP \)-odd part of the effective potentials \( V_{ab} \) and \( W_{ab} \) that are of higher order in the perturbation. Here we assume the validity of the perturbative approximation, within which the effects of the back reaction and rescattering are determined in the same way as in the case without \( CP \) violation. For simplicity, we use the densities in the tenth interval when we evaluate the total densities of the generated particles and charges, since 10 is a typical number of the oscillations before the preheating ends.
We now discuss the numerical results. First, neglecting the back reaction, we studied the time evolution of the densities for $\theta_0 = 10^{-3}$, $l_1 = 0.01$ and $l_2 = 0.02$. Since $\beta_k$ in (3.12) behaves as $|\beta_k| \simeq e^{-\pi \kappa^2}$, the modes with $\kappa$ satisfying $\kappa \lesssim 1/\sqrt{\pi} \simeq 0.56$ are expected to belong to the first resonance band. The time evolution of $n_k$ and $j_{1k}$ for several values of $\kappa$ are displayed in Figs. 1 and 2. As expected, the densities for $\kappa \lesssim 0.5$ grow exponentially, and $|j_{1k}^j| \sim 10^{-4} n_k^j$ for this choice of the parameters.

We estimate the total densities of the generated particles and charges by integrating $n_k^j$ and $j_{1k}^j$ at $j = 10$ over $k$:

$$n = \int d^3k n_k = 8\sqrt{2\pi} m^3 q^{3/4} \int_0^\infty dk \kappa^2 n_k,$$

$$j_1 = 8\sqrt{2\pi} m^3 q^{3/4} \int_0^\infty dk \kappa^2 j_{1k},$$

$$j_2 = 8\sqrt{2\pi} m^3 q^{3/4} \int_0^\infty dk \kappa^2 j_{2k}. \quad (3.22)$$

We display the behavior of the integrands at $j = 10$ as functions of $\kappa$ for several
values of $\theta_0$ in Fig. 3. We see that, except for the case $\theta_0 = \pi$, the generated particle numbers are not affected by the $CP$ violation, while the generated charge is nearly proportional to $\theta_0$. As seen from the $\kappa$ dependence of the integrands of (3.22) in Fig. 3, the second resonance band exists near $\kappa = 0.8$, but its contribution is about five orders smaller than that from the first resonance band. We performed the calculation up to $\kappa = 10$ and found several higher resonance bands, whose contributions are negligible. The values of the integrals in (3.22) are listed in Table I.

We also calculated the total densities for various values of the coupling constants $l_1$ and $l_2$ for $\theta_0 = \pi$. As shown in Table II, there are no regularities in dependence of the total densities on the coupling constants.

Table I. The integrals appearing in the total particle number density and charge density (3-22) for several values of $\theta_0$.

<table>
<thead>
<tr>
<th>$\theta_0$</th>
<th>$\int d\kappa \kappa^2 n_k$</th>
<th>$\int d\kappa \kappa^2 j_{1k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-3}$</td>
<td>130.5096</td>
<td>$-1.609334 \times 10^{-2}$</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>130.5156</td>
<td>$-1.544579 \times 10^{-1}$</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>131.1163</td>
<td>$-1.537716$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>990.7411</td>
<td>$-50.84228$</td>
</tr>
</tbody>
</table>

Table II. The integrals appearing in the total particle number density and charge density (3-22) for several values of $l_1$ and $l_2 = 2l_1$ with $\theta_0 = \pi$.

<table>
<thead>
<tr>
<th>$l_1$</th>
<th>$\int d\kappa \kappa^2 n_k$</th>
<th>$\int d\kappa \kappa^2 j_{1k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td>1.071055 $\times 10^5$</td>
<td>2.456463 $\times 10^2$</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>6.773442 $\times 10^4$</td>
<td>$-2.316711 \times 10^3$</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>9.907411 $\times 10^2$</td>
<td>$-5.084228 \times 10^1$</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>1.656527 $\times 10^3$</td>
<td>$-2.069592 \times 10^2$</td>
</tr>
</tbody>
</table>

§4. Discussion

Since all particles existing presently are thought to have been created after the reheating process in the inflationary scenario, and since this process is far from equilibrium, it is natural to believe that the asymmetry between matter and antimatter was also generated in this era. We studied charge generation during the preheating, which is viewed as a particle creation in the oscillating background.
If the scale factor of the FRW universe changes from $R_{\text{preheating}}$ to $R_{\text{th}}$ just after thermalization, the charge-to-entropy ratio will be given by

$$\frac{j_1}{s} = \frac{8\sqrt{2\pi} m^3 q^{3/4}}{2\pi^2} \int_0^\infty dk \frac{\kappa^2 j_{1k}}{g_s s T_{\text{th}}^3} \left( \frac{R_{\text{preheating}}}{R_{\text{th}}} \right)^3,$$

where $g_s s$ counts the degrees of freedom contributing to the entropy and $T_{\text{th}}$ is the temperature just after thermalization. To this point, we have not specified what the charge is, but we expect it to be conserved after the preheating process. One may think of it as the baryon or lepton number, regarding the $\chi$ field as the scalar partner of the quarks or leptons. If the preheating occurs before the electroweak phase transition, the generated lepton number is converted to the baryon number through the sphaleron process. If the charge is a weak hypercharge, the equilibrium state after thermalization with the hypercharge excess will become a baryon-rich state, as predicted by the theory of electroweak baryogenesis. In any case, our next task will be to construct a realistic model, including at least two complex scalars, which describes the charge generation based on the mechanism studied here and to evaluate the baryon asymmetry produced by that model.

Although we have considered a coherently oscillating field as the inflaton, the present formalism may be applicable to the Affleck-Dine (AD) scalar, which rotates coherently in the internal space with a constant frequency if we neglect the damping due to the expansion of the universe. The AD scalar, which is a classical squark or slepton field, is thought to decay perturbatively into quarks and leptons to leave nonzero $B$ or $L$. This situation is similar to that of the conventional reheating process and one may expect that there is also a process similar to preheating in the AD mechanism.

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