On the Relation between Non-Local Urmaterie Field and Irreducible Local Field

Y. Ohnuki and O. Hara

Institute of Theoretical Physics, Nagoya University

November, 30, 1953

It was verified by many authors\(^{1,2}\) that Yukawa's non-local field\(^{3}\) was a superposition of irreducible local fields with various spin. In their investigations, Yukawa's third equation played an important role.

Recently O. Hara et. al\(^{[1,4,5]}\) proposed a non-local "Urmaterie" field in order to describe elementary particles in unified way. They started from the following two fundamental equations\(^{*}\)

\[
\begin{align*}
\langle \partial^2/\partial X_\mu \partial X_\mu - \mathbf{M}^2/\mathbf{k}^2 \rangle U(X_\mu, r_\mu) &= 0, \\
(r_\mu^a r_\mu^a - \mathbf{k}^2) U(X_\mu, r_\mu) &= 0.
\end{align*}
\]

(1)

In this case, \(\mathbf{M}^2\) is not commutable with \(r_\mu \partial/\partial X_\mu\), and Yukawa's third equation can not be adopted. It is shown in this note, however, that the free scalar "Urmaterie" field can be regarded also as a superposition of local fields with various spin and rest mass which satisfy Fierz's\(^{7}\) equations.

Neglecting normalization constants, the eigenfunction of the internal motion which belongs to the eigenvalue \(S(S+1)\) and \(m^2\) of \(S^2\) and \(\mathbf{M}^2\) is given by

\[
\psi_{s, m, n} = \frac{1}{(r_\mu r_\mu - \mathbf{k}_\mu \mathbf{k}_\mu)^2} \frac{1}{(r_\mu r_\mu)^{1/2}}
\]

\[
\times \int_{-\pi}^{\pi} du \left( i\mu r_\mu + \mathbf{i}(r_\mu r_\mu) \cos u + i\alpha r_\mu \sin u \right) e^{i\mu u},
\]

(2)

where \(f_{s, m}\) is determined to be a product of Jacobi's polynomials and some algebraic functions as an eigenfunction of \(\mathbf{M}^2\) in the rest system of the center of mass, \(\mathbf{k}_\mu\) the energy-momentum vector of the external motion, and \(\alpha_{\mu \nu}\) are coefficients of Lorentz-transformation which transform \(\mathbf{k}_\mu\) into rest.

\[
\alpha_{\mu \nu} = \begin{cases} 
0 & \text{for } \mu = 1, 2, 3, \\
\frac{i(m\mu\lambda)}{\lambda} & \text{for } \mu = 4.
\end{cases}
\]

(3)

Of course \(\alpha_{\mu \nu}\) satisfy

\[
\alpha_{\mu \lambda} \alpha_{\lambda \nu} = \delta_{\mu \nu}.
\]

(4)

In (2) Lorentz-invariant parameter \(n\) is introduced other than \(s\) and \(m\) to distinguish \(2r+1\) independent eigenfunction belonging to the spin eigenvalue \(s\), and the last factor is a four dimensional generalization of spherical harmonics which in the rest system of the center of mass, is reduced to

\[
P_{s, m}^{(\mu)}(\cos \theta)e^{i\mu \pi} = \text{const} \int_{-\pi}^{\pi} du \left( r_\mu + i r_\mu \cos u + ir_\mu \sin u \right) e^{i\mu u}.
\]

Expanding Fourier coefficients of the Urmaterie field by these eigenfunctions,

\[
U(X_\mu, r_\mu) = \int u(\delta_{\mu \nu}, r_\mu) e^{i\mathbf{k}_\mu X_\mu} (d\mathbf{k}),
\]

\[
u(\mathbf{k}_\mu, r_\mu) = \sum \Delta(\mathbf{k}_\mu; \nu, s, m) \delta(\mathbf{r}_\mu - k^2) \left( r_\mu r_\mu - \mathbf{k}_\mu \mathbf{k}_\mu \right)^{\lambda/2} f_s \left( -\mathbf{k}_\mu \mathbf{k}_\mu \mathbf{r}_\mu \right)^{1/2}
\times \int_{-\pi}^{\pi} du \left( i\mu r_\mu + \mathbf{i}(r_\mu r_\mu) \cos u + i\alpha r_\mu \sin u \right) e^{i\mu u},
\]

(5)

and picking up the coefficient of \(r_\lambda r_\mu r_\nu\) from

\[
\sum_{|\lambda, \mu, \nu|} \Delta(\mathbf{k}_\mu; \nu, s, m) \int_{-\pi}^{\pi} du \left( i\mu r_\mu + \mathbf{i}(r_\mu r_\mu) \cos u + \mathbf{i}(r_\mu r_\mu) \sin u \right) e^{i\mu u},
\]

(6)

we see that it behaves as a symmetrical tensor contrary to \(r_\lambda r_\mu r_\nu\), since \(\nu(\mathbf{k}_\mu, \nu, s, m)\) is scalar. Denoting it as \(\Delta(\mathbf{k}_\mu; \nu, s, m)\), it is given by

\[
A_{\lambda \mu \nu} \cdots(\mathbf{k}_\mu; \nu, s, m) = \sum_{|\lambda, \mu, \nu|} \int_{-\pi}^{\pi} du \Delta(\mathbf{k}_\mu; \nu, s, m) e^{i\mu u}.
\]
Letters to the Editor

\[
\times (a_{11} + ia_{11} \cos \nu + ia_{13} \sin \nu)^n \\
\times (a_{12} + ia_{12} \cos \nu + ia_{23} \sin \nu)^n \\
\times (a_{13} + ia_{13} \cos \nu + ia_{33} \sin \nu)^n \\
\times (a_{14} + ia_{14} \cos \nu + ia_{34} \sin \nu)^n 
\], \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7)
\]

where \(a, \beta, \gamma\) and \(\delta\) mean numbers of 1, 2, 3, and 4 appearing in suffixes of \(A_{\mu \nu} \ldots (\beta_{\nu} ; s, m)\) respectively. Using (1), (4) and (7), we get after some calculations

\[
\begin{align*}
(k_\mu k_\mu + (m | \mu \rangle \langle \mu | m) A_{\mu \nu} \ldots (k_\mu ; s, m) &= 0, \\
A_{\lambda \nu} \ldots (k_\mu ; s, m) &= 0, \\
A_{\lambda \nu} \ldots (k_\mu ; s, m) &= 0.
\end{align*}
\]

(8)

These are nothing but the equations of motion and the supplementary conditions for particles with spin \(s\) and rest mass \((m | \mu \rangle \langle \mu | m)\) as given by Fierz\(^5\). It is clear from above deduction that (8) is a direct consequence of the fact that the eigenfunction of \(S^2\) is essentially spherical harmonics, and therefore is closely related to the rotation of a rigid sphere. Main features of Yukawa's original non-local theory are thus maintained in our theory of the Urmaterie field.

Equations (3) and (8) show that the Urmaterie field is equivalent to a superposition of various local fields with definite spin and rest mass, the value of which are determined by the internal motion described by eigenfunctions accompanying to each of them. Such a structure of the Urmaterie field suggests a way for its quantization, since the quantization is a procedure to reproduce particle aspect from that of the wave, and particles that appear in our observation seem to have definite spin and rest mass at least in the case of no interaction. Thus, it would be reasonable to expect that the quantization of the (free) Urmaterie field can be achieved by quantizing those part of it which corresponds to local fields. Eigenfunctions of the internal motion will, on the other hand, be responsible to the law of interaction between them. Although the concrete from of this law is not known at the present stage, it is easy to foresee, for example by substituting (5) into the \(S\)-matrix given by Yukawa\(^6\), that it leads to a non-local interaction with form factors which are composed from these eigenfunctions. Thus, in the theory of the Urmaterie field, the form of form factors is determined uniquely by the structure of participating particles. As emphasized by Yukawa\(^6\), this is one of the great advantages of the theory of this type over the conventional theory of non-local interaction, where we can never expect such principle to be given.

Details will be published in a near future with related problems.

4) O. Hara and T. Marumori, Prog. Theor. Phys. 9 (1953), 559.

Some Remarks on the Mass Spectrum and Non-local Interaction

H. Goto

Faculty of Engineering, Gifu University

November, 30, 1953

Recently, Yukawa\(^11\) proposed an interesting attempt which introduced the mass spectrum of the elementary particles by non-local field theory. This attempt seems to be an important step towards the construction of a consistent field theory free from "divergent difficulties". But this theory includes two difficulties in its formalism.

In the first time, the mass spectrum induced by this has the infinite degeneracy, or the finite degeneracy.

In the next time, the non-local interaction obtained by integrating with respect to the internal coordinate destroys the relativistic requirements. As to this point, though Utiyama has pointed out to treat by unitary trick, this method could not obtain the sufficient results.

The author tried to overcome these difficulties by modification of the fundamental equation originated by Yukawa.

As an example of the fundamental equation, let us consider the following one for the scalar field,